نام و نام خانوادگی: [illegible] .... پاسخنامه تشریحی تکلیف شماره ۲۳ ...... کلاس دوازدهم ریاضی A ....

$g(x) = f(\tan^2 x + \sqrt{2}\cos x) \to g'(x) = \left(2(1+\tan^2 x)\tan x - \sqrt{2}\sin x\right) f'(\tan^2 x + \sqrt{2}\cos x)$ $\underline{x=\frac{\pi}{4}}$ $g'(\frac{\pi}{4}) = \left(2(1+1)\times 1 - \frac{\sqrt{2}\times\sqrt{2}}{2}\right) f'(1 + \sqrt{2}\times\frac{\sqrt{2}}{2})$ $\sqrt{3} = (4-1) f'(2) \to \boxed{f'(2) = \frac{\sqrt{3}}{3}}$	۱
$g(x) = \frac{2}{2-\cos x} \to g'(x) = \frac{-2\sin x}{(2-\cos x)^2} \to g'(\frac{\sqrt{\pi}}{6}) = \frac{+2\times\frac{1}{2}}{(2+\frac{\sqrt{3}}{2})^2} = \frac{4}{16+3+8\sqrt{3}} = \frac{4}{19+8\sqrt{3}}$ $f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} \to f'(x) = \frac{(3^2 x - \sin x \times \cos^2 x)(4-\cos^2 x) - (2\sin x \times \cos x)(8+\cos^3 x)}{(4-\cos^2 x)^2}$ $f'(\frac{\sqrt{\pi}}{6}) = \frac{(3\times -(-\frac{1}{2})\times(\frac{3}{4}))(4-\frac{3}{4}) - (\frac{\sqrt{3}}{2})\times(8-\frac{3\sqrt{3}}{8})}{(4-\frac{3}{4})^2} = \frac{(\frac{9}{8}\times\frac{13}{4}) - (4\sqrt{3}-\frac{9}{16})}{\frac{169}{16}} = \frac{\frac{117}{32} - 4\sqrt{3} + \frac{9}{16}}{\frac{169}{16}}$ $\frac{9}{8}$, $\frac{13}{4}$, $\frac{117}{32}$, $(4-\frac{3}{4})^2 \to \frac{13^2}{4^2} = \frac{169}{16}$	۲
$f'(x) \to \lim_{x\to\frac{3}{4}^+} \frac{(4x-3)\sqrt{ax} - 0}{x-\frac{3}{4}} = 4\sqrt{ax} = 4\sqrt{\frac{3a}{4}} = 2\sqrt{3a}$ $\to \lim_{x\to\frac{3}{4}^-} \frac{-(4x-3)\sqrt{ax} - 0}{x-\frac{3}{4}} = -4\sqrt{ax} = -4\sqrt{\frac{3a}{4}} = -2\sqrt{3a}$ $2\sqrt{3a} - (-2\sqrt{3a}) = 2\sqrt{6} \to 4\sqrt{3a} = 2\sqrt{6}$ $\sqrt{3a} = \frac{\sqrt{6}}{2} \to 3a = \frac{6}{4}$ $\to \boxed{a = \frac{1}{2}}$	۳
$f'(4) = -a \quad , \quad f(4) = 2 - 4a$ $f(4) + f'(4) = -1 \to 2 - 4a - a = -1 \to 5a = 3 \to a = \frac{3}{5}$ $\boxed{f(4) = -0/6}$ $\frac{\frac{135}{32} - 4\sqrt{3}}{\frac{169}{16}} \to \frac{(135-128\sqrt{3})16}{2\times 32\times 169} = \frac{135-128\sqrt{3}}{338}$ $f'(\frac{\sqrt{\pi}}{6}) - 2g'(\frac{\sqrt{\pi}}{6}) = \frac{135-128\sqrt{3}}{338} - \frac{8}{19+8\sqrt{3}} = \frac{-169}{338} = \boxed{-\frac{1}{2}}$	۴
$y = \frac{x+5}{7}$ $y = \frac{ax-1}{3x+1}$ $\to \frac{x+5}{7} = \frac{ax-1}{3x+1} \to 3x^2 + 16x + 5 = 7ax - 7$ وقتی دو نمودار بر هم مماس باشند یعنی معادله تلاقی آنها ریشه مضاعف دارد $\leftarrow \Delta = 0$ $\to 3x^2 + (16-7a)x + 12 = 0$ ← چون حل تلاقی ناحیه اول باید باشد $\Delta = 0 \to (16-7a)^2 - 4\times 12\times 3 = 0 \to (16-7a)^2 = 144 \to$ $16-7a = 12 \to 7a = 4 \to \boxed{a = \frac{4}{7}}$ ✗ $16-7a = -12 \to 7a = 28 \to \boxed{a = 4}$ ✓	۵

← جواب قابل قبول

$f(2)=0 \rightarrow 8+2a-b=0 \rightarrow 8-24-b=0 \rightarrow b=-16$

$f'(2)=0 \rightarrow 12+a=0 \rightarrow a=-12$

$-16-(-12)=\boxed{-4}$ ۶

$2^5 \quad 2^{\frac{1}{2}} = 2^{5/5}$

$f(x)=\sin^n(x^2) \quad , \quad \lim \dfrac{f(x)f'(x)}{(1-\cos x)^m} = 32\sqrt{2} \rightarrow \lim_{x\to 0} \dfrac{2nx\cos(x^2)\sin^{2n-1}(x^2)}{(1-\cos x)^m}$ $\quad x^{4n-1}$

$\downarrow$

$f'(x)=n\sin^{n-1}(x^2)\times\cos(x^2)\times 2x \quad \xrightarrow[1-\cos x \sim \frac{x^2}{2},\ x\to 0]{\sin x \sim x,\ x\to 0} \dfrac{(2nx)(1)(x^2)^{2n-1}}{\frac{x^{2m}}{2^m}} = \dfrac{(2n\times 2^m)(x)(x^{4n-2})}{x^{2m}} = 2^{5/5}$

$n\cdot 2^{m+1} = 2^{5/5}$

$n=2 \rightarrow 2^{m+2} = 2^{5/5} \rightarrow \boxed{m=3/5}$ (۲)

$x^{2m}=x^{4n-1} \rightarrow 2m=4n-1$ (۱) $\xrightarrow{\text{اعداد جدید}}$ آمده در (۲) در آن صدق می‌کنند $\rightarrow 2m+n=7+2=\boxed{9}$

$y=\dfrac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow \sqrt{x}y-y=\sqrt{x}+1 \rightarrow \sqrt{x}y-\sqrt{x}=y+1 \rightarrow \sqrt{x}(y-1)=y+1$

$\rightarrow \sqrt{x}=\dfrac{y+1}{y-1} \rightarrow x=\left(\dfrac{y+1}{y-1}\right)^2 \rightarrow f^{-1}(x)=\left(\dfrac{x+1}{x-1}\right)^2$

$f^{-1'}(x)=2\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{(x-1)-(x+1)}{(x-1)^2}\right) \Rightarrow f^{-1'}(2)=2(3)(-2)=\boxed{-12}$ ۸

$[-1,0]$ آهنگ تغییر متوسط در $\rightarrow \dfrac{f(0)-f(-1)}{0-(-1)} = \dfrac{1-(8(1-a))}{1} = 8a-7=-11 \rightarrow a=-\frac{1}{2}$

$f(x)=(x^2+1)^3\left(-\frac{x}{2}+1\right)$

$f'(x)=3(2x)(x^2+1)^2\left(-\frac{x}{2}+1\right)+-\frac{1}{2}(x^2+1)^3$ ۹

$f'(1)=3\times 2\times 4\times\frac{1}{2}-4=12-4=\boxed{8}$

$y=\sin x\cos 2x \xrightarrow[\text{آهنگ}]{[\frac{\pi}{4},\frac{\pi}{2}]} \dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}} = \dfrac{-1-0}{\frac{\pi}{4}} = \dfrac{-4}{\pi}$ (۱)

$y=\sin^4 x-\cos^4 x \xrightarrow[\text{آهنگ}]{[\frac{\pi}{4},\frac{\pi}{2}]} \dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}} = \dfrac{1-0}{\frac{\pi}{4}} = \dfrac{4}{\pi}$ (۲) ۱۰

$\dfrac{(1)}{(2)} = \dfrac{-\frac{4}{\pi}}{\frac{4}{\pi}} = \boxed{-1}$