① $g(x)=f(\tan^2 x+\sqrt{2}\cos x)$, $g'\left(\frac{\pi}{4}\right)=\sqrt{2}$ مشتق ضمنی $\Rightarrow g'(x)=2\left(\left(1+\tan^2 x\right)\tan x-\sqrt{2}\sin x\right)f'(\tan x+\sqrt{2}\cos x)\Rightarrow$

$\Rightarrow g'\left(\frac{\pi}{4}\right)=\left(2(2)(1)-\frac{\sqrt{2}\times\sqrt{2}}{2}\right)f'\left(1+\frac{\sqrt{2}+\sqrt{2}}{2}\right)\Rightarrow\sqrt{2}=3f'(2)\Rightarrow f'(2)=\frac{\sqrt{2}}{3}$

② $f(x)=\frac{8+\cos^3 x}{4-\cos^2 2x}$, $g(x)=\frac{2}{2-\cos x}$, $g'(x)=\frac{-2\sin x}{(2-\cos x)^2}\Rightarrow g'\left(\frac{2\pi}{3}\right)=\frac{4}{19+8\sqrt{3}}$

$f'(x)=\frac{\left(-3\sin x\cos^2 x\right)\left(4-\cos^2 2x\right)-\left(2\sin x\cos 2x\right)\left(8+\cos^3 x\right)}{\left(4-\cos^2 2x\right)^2}\Rightarrow f'\left(\frac{\pi}{3}\right)=\frac{\left(\frac{9}{8}\times\frac{13}{4}\right)-\left(4\sqrt{3}-\frac{9}{16}\right)}{\frac{169}{16}}$

$f'\left(\frac{\pi}{3}\right)-2g'\left(\frac{\pi}{3}\right)=\frac{117-128\sqrt{3}}{338}-\frac{8}{19+8\sqrt{3}}=\frac{-169}{338}=-\frac{1}{2}$

③ $f(x)=|4x-3|\sqrt{ax}\Rightarrow f'(x)$

$\lim_{x\to\left(\frac{3}{4}\right)^+}\frac{(4x-3)\sqrt{ax}-0}{x-\frac{3}{4}}=4\sqrt{ax}=4\frac{\sqrt{3a}}{2}=2\sqrt{3a}$

$\lim_{x\to\left(\frac{3}{4}\right)^-}\frac{-(4x-3)\sqrt{ax}-0}{x-\frac{3}{4}}=-4\sqrt{ax}=-4\frac{\sqrt{3a}}{2}=-2\sqrt{3a}$

$2\sqrt{3a}-\left(-2\sqrt{3a}\right)=2\sqrt{9}\Rightarrow\sqrt{3a}=\frac{\sqrt{9}}{2}\Rightarrow 3a=\frac{9}{4}\Rightarrow a=\frac{1}{4}$

④ $f(x)+f'(x)=-1$, $f'(x)=-a$, $f(x)=2-2x a$, $f(2)+f'(2)=-1\Rightarrow 2-4a-a=-1\Rightarrow a=\frac{3}{5}\Rightarrow f'(2)=0.6$

⑤ $y=\frac{ax-1}{3x+1}$, $y-x=5\Rightarrow y=\frac{x+5}{1}$, $\frac{x+5}{1}=\frac{ax-1}{3x+1}\Rightarrow 3x^2+14x+5=7ax-7\Rightarrow$

$\Rightarrow 3x^2+(14-7a)x+12=0\Rightarrow\Delta=0\Rightarrow(14-7a)^2-4(12)(3)=0\Rightarrow\begin{cases}14-7a=12\Rightarrow 7a=2\Rightarrow a=\frac{2}{7} & \text{غ ق ق}\\ 14-7a=-12\Rightarrow 7a=26\Rightarrow a=4 & \text{ق ق}\end{cases}$

⑥ $f(x)=x^3+ax-b$, $f(2)=0\Rightarrow 8+2a-b=0\Rightarrow b=-14$, $f'(2)=0\Rightarrow 12+a=0\Rightarrow a=-12$

$(-14)-(-12)=-2$

⑦ $f(x)=\sin^n(x^2)$, $\lim_{x\to 0}\frac{f(x)f'(x)}{(1-\cos x)^m}=2\sqrt{2}$, $f'(x)=n\sin^{n-1}(x^2)\times\cos(x^2)\times 2x$

$\lim_{x\to 0}\frac{f(x)f'(x)}{(1-\cos x)^m}=\frac{2n\,x\cos(x^2)\sin^{2n-1}(x^2)}{(1-\cos x)^m}=\frac{(2n)(1)(x^2)^{2n-1}}{\frac{x^{2m}}{2^m}}=\frac{(2n2^m)(x)(x^{4n-2})}{x^{2m}}=\frac{n\times 2^{m+1}\times x^{4n-1}}{x^{2m}}=2$ ω،ه

$2^{m+1}\times n=2^{\omega,\delta}\Rightarrow n=2$ ، $m=2,5$, $2m+n=7+2=9$

⑧ $f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1}\Rightarrow y=\frac{\sqrt{x}+1}{\sqrt{x}-1}\Rightarrow$

$\sqrt{x}\,y-y=\sqrt{x}+1\Rightarrow\sqrt{x}(y-1)=y+1\Rightarrow\sqrt{x}=\frac{y+1}{y-1}\Rightarrow f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2$

$\left(f^{-1}\right)'(x)=2\left(\frac{x+1}{x-1}\right)\left(\frac{(x-1)-(x+1)}{(x-1)^2}\right)\Rightarrow$

$\left(f^{-1}\right)'(2)=2\,(-2)=-12$

⑨ $f(x)=(x^2+1)^3(ax+1)$, $\frac{f(0)-f(-1)}{0-(-1)}=\frac{1-(8(1-a))}{1}=8a-7=-11\Rightarrow a=-\frac{1}{2}$

$f'(x)=3(2x)(x^2+1)^2\left(-\frac{x}{2}+1\right)+\left(-\frac{1}{2}\right)(x^2+1)^3$, $f'(1)=(3)(2)(1)\left(\frac{1}{2}\right)-4=12-4=8$

⑩ $y_1=\sin x\cos 2x\xrightarrow{\text{آهنگ آن}}\frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{4}\right)}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-1-0}{\frac{\pi}{4}}=\frac{-4}{\pi}$, $y_2=\sin^4 x-\cos^4 x\xrightarrow{\text{آهنگ}}\frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{4}\right)}{\frac{\pi}{2}-\frac{\pi}{4}}=$

$=\frac{1-0}{\frac{\pi}{4}}=\frac{4}{\pi}$, $\frac{y_1}{y_2}=\frac{\frac{-4}{\pi}}{\frac{4}{\pi}}=-1$

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