۱۹٫۵

نام و نام خانوادگی ......آترین جلیلی...... پاسخنامه تشریحی تکلیف شماره ...۲۴... کلاس ..دوازدهم تجربی.. A

۱	$f(3)=5$, $f(0)=1 \Rightarrow m=\frac{5-1}{3-0}=\frac{4}{3} \rightarrow f'(3)=\frac{4}{3}$ ✓ (۲)
۲	$f(-1)=1$, $f(2)=2 \Rightarrow m=\frac{2-1}{2-(-1)}=\frac{1}{3} \Rightarrow y-1=\frac{1}{3}(x+1) \rightarrow y=\frac{1}{3}x+\frac{4}{3}$ $\rightarrow \sqrt{ax-1}=\frac{1}{3}x+\frac{4}{3} \rightarrow ax-1=\frac{(x+4)^2}{9}\ (I)$, $f'(x)=\frac{a}{2\sqrt{ax-1}}=\frac{1}{3} \rightarrow 9a^2=4(ax-1)$ $\xrightarrow{II} ax-1=\frac{9a^2}{4} \xrightarrow{(I)} \frac{(x+4)^2}{9}=\frac{9a^2}{4} \Rightarrow x+4=\frac{9a}{2} \Rightarrow x=\frac{9a}{2}-4$ $\xrightarrow{II} a\left(\frac{9a}{2}-4\right)-1=\frac{9a^2}{4} \Rightarrow a=2 \Rightarrow f(5)=\sqrt{5(2)-1}=3$ ✓ (۲)
۳	$y=\frac{x^2+mx+1}{x+3} \rightarrow y'=\frac{x^2+6x+3m-1}{(x+3)^2} \rightarrow f'(1)=\frac{6+3m}{16}$ $4y-3x=n \rightarrow y=\frac{3}{4}x+\frac{n}{4} \rightarrow y'=\frac{3}{4}$ $\Rightarrow \frac{6+3m}{16}=\frac{3}{4} \rightarrow m=2$ $y=\frac{1+2+1}{1+3}=1 \rightarrow A(1,1) \rightarrow 4y-3x=n \rightarrow 4(1)-3(1)=n \rightarrow n=1$ $\Rightarrow m+n=2+1=3$ ✓ (۲)
۴	$3g'\left(\frac{5\pi}{3}\right)-f'\left(\frac{5\pi}{3}\right)=(3g-f)'=\left(\frac{9}{3+\sin x}-\frac{27-\sin^3 x}{9-\sin^2 x}\right)'$ $=\left(\frac{9}{3+\sin x}-\frac{(3-\sin x)(9+3\sin x+\sin^2 x)}{(3-\sin x)(3+\sin x)}\right)'=\left(\frac{9-9-3\sin x-\sin^2 x}{3+\sin x}\right)'=\left(-\frac{(\sin x)(\sin x+3)}{\sin x+3}\right)'$ $=-(\sin x)'=-\cos x \xrightarrow{x=\frac{5\pi}{3}} -\cos\left(\frac{5\pi}{3}\right)=-\frac{1}{2}$ ✓ (۲)
۵	$g'(\sqrt[5]{3})\,f'(g(\sqrt[5]{3}))=(f\circ g)'(\sqrt[5]{3})$ $f(g(x))=f\left(\frac{1}{2x^5}\right)=-\frac{1}{\sqrt[5]{2\left(\frac{1}{2x^5}\right)}}=\frac{-1}{x^{-1}}=-x \rightarrow (f\circ g)'(x)=(-x)'=-1$ ✓ (۲)

$f(x)=\left(\frac{\sin x-1}{\sin x+1}\right)^2 \to f'(x)=2\left(\frac{\sin x-1}{\sin x+1}\right)\left(\frac{2\cos x}{(\sin x+1)^2}\right) \to f'(0)=2(-1)(2)=-4$

$f(x)=xg(x)+1 \to g(x)=\frac{f(x)-1}{x}$

$\lim_{x\to 0} g(x)=\lim_{x\to 0}\frac{f(x)-1}{x}=f'(0) \longrightarrow \lim_{x\to 0} g(x)=f'(0)=-4$

(۲)

$y=x^2+1 \xrightarrow{\text{قرینه نسبت به محور } x} y=-x^2-1 \xrightarrow{y=k} -x^2-1=k \to x^2=-1-k \to x=\pm\sqrt{-1-k}\ ,\ k<-1$

$y'=-2x \to \begin{cases} m_1=-2\sqrt{-1-k}\ ,\ x_1=+\sqrt{-1-k} \\ m_2=2\sqrt{-1-k}\ ,\ x_2=-\sqrt{-1-k} \end{cases} \xrightarrow{\text{بر هم عمودند}} m_1\times m_2=-1$

$\to -4(-1-k)=-1 \to 1+k=\frac{-1}{4} \to k=\frac{-5}{4}$ فاصله خط d از مبدا مختصات $=|k|=\frac{5}{4}$

(۲)

$f(x)=2(x)^{\frac12}(4x^2+3) \to f'(x)=2\left[\frac12 x^{-\frac12}(4x^2+3)+x^{\frac12}\cdot 8x\right]=2\left[\frac{4x^2+3}{2\sqrt{x}}+8x^{\frac32}\right]$

$=\frac{4x^2+3}{\sqrt{x}}+16x^{\frac32}$, $f(a)=ma \to 2\sqrt{a}(4a^2+3)=ma \to m=\frac{2\sqrt{a}(4a^2+3)}{a}=\frac{2(4a^2+3)}{\sqrt{a}}$ (I)

$f'(a)=m \to \frac{4a^2+3}{\sqrt{a}}+16a^{\frac32}=m \xrightarrow{(I)} \frac{4a^2+3}{\sqrt{a}}+16a^{\frac32}=\frac{2(4a^2+3)}{\sqrt{a}} \to 16a^{\frac32}=\frac{4a^2+3}{\sqrt{a}}$

$\to 16a^2=4a^2+3 \to a=\frac12 \to m=\frac{2(4a^2+3)}{\sqrt{a}}=\frac{2(4\cdot\frac14+3)}{\sqrt{\frac12}}=\frac{8}{\frac{1}{\sqrt2}}=8\sqrt2$

(۲)

$f(x)=\frac{\sqrt{x}}{-2x^2+x+1}$, $y=mx \to f(A)=mA \to m=\frac{f(A)}{A}=f'(A)$

$\frac{f(x)}{x}=\frac{\sqrt{x}}{x(-2x^2+x+1)}=\frac{1}{\sqrt{x}(-2x^2+x+1)} \to f'(x)=\frac{6x^2-x+1}{2\sqrt{x}(-2x^2+x+1)^2} \xrightarrow{f'(A)=\frac{f(A)}{A}} \frac{6A^2-A+1}{2\sqrt{A}(-2A^2+A+1)^2}=\frac{1}{\sqrt{A}(-2A^2+A+1)}$

$\to \begin{cases} A=\frac12 \\ A=\frac{-1}{5} \end{cases} \xrightarrow{x\geq 0} A=\frac12 \to f(\frac12)=\frac{\sqrt{\frac12}}{-2(\frac12)^2+\frac12+1}=\frac{1}{\sqrt2}=\frac{\sqrt2}{2}$

(۲)

$g(\frac{\sqrt5}{2})=\frac{1}{\sqrt{\frac54-1}}=\frac{1}{\sqrt{\frac14}}=2 \to f\circ g(\frac{\sqrt5}{2})=f(2)$ $\left(f\circ g(\frac{\sqrt5}{2})\right)'^{-}=g'(\frac{\sqrt5}{2})^{-}\times f'(g(\frac{\sqrt5}{2})^{-})$

$x\to 2^- \Rightarrow [x]=1 \to f(x)=x^3 \to f'_-(2)=3x^2=3(2)^2=12$

$g(x)=(x^2-1)^{-\frac12} \to g'(x)=\frac{-x}{(x^2-1)^{\frac32}} \to g'(\frac{\sqrt5}{2})=-4\sqrt5$

$(f\circ g)'(x)=f'(g(x))\cdot g'(x) \to (f\circ g)'(\frac{\sqrt5}{2})=12\times(-4\sqrt5)=-48\sqrt5 \to \frac{(f\circ g)'_-(\frac{\sqrt5}{2})}{-48\sqrt5}=1$

(۱٫۵)

$g(x)=(x^2-1)^{-\frac12} \to g'(x)=\frac{-1}{2}(x^2-1)^{-\frac32}\times 2x \to g'(\frac{\sqrt5}{2}^-)=\frac{1}{\sqrt{(\frac{\sqrt5}{2})^2-1}}=\frac{1}{\sqrt{(\frac14)^-}}=\frac{1}{(\frac12)^-}=2^+$

$f'(2^+)=((2x)^3)'=(8x^3)'=24x^2=24\times 4 \to g'(\frac{\sqrt5}{2}^-)\times f'(g(\frac{\sqrt5}{2})^-)=-4\sqrt5\times 4\times 24 \to \frac{24\times 4\times(-4\sqrt5)}{-4\sqrt5}=$