مجموعه دعوت‌نامه C / دوازدهم تجربی A ۱۸

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$$\left.\begin{matrix}(0,1)\\(3,5)\end{matrix}\right\} m=\frac{5-1}{3-0}=\frac{4}{3}$$

$$\Rightarrow y=\frac{4}{3}x+1 \Rightarrow y'=\frac{4}{3}=f'(x)$$

$$\Rightarrow \boxed{f'(3)=\frac{4}{3}}$$

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شیب $m=\dfrac{2-1}{2-(-1)}=\dfrac{1}{3} \Rightarrow y=\dfrac{1}{3}x+\dfrac{4}{3}$

$$\Rightarrow \frac{1}{3}x+\frac{4}{3}=\sqrt{ax-1} \Rightarrow \frac{(4+x)^2}{9}=ax-1$$

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$$\Rightarrow (x+4)^2=9ax-9$$

$$\Rightarrow x^2-(8-9a)x+25=0 \Rightarrow (x-5)^2$$

$$\Rightarrow 8-9a=\pm 10 \Rightarrow a=-\frac{2}{9}, 2$$

$$\Rightarrow f(5)=\sqrt{2(5)-1}=\boxed{3}$$

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$$4y-3x=n \Rightarrow m=-\frac{(-3)}{4}=\frac{3}{4}$$

$$y=\left(1,\frac{2+m}{4}\right) \Rightarrow y'=\frac{(2+m)(4)-(2+m)}{(4)^2}=\frac{3}{4}$$

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$$\Rightarrow \boxed{m=2}$$

$$y=\frac{3}{4}x+\frac{n}{4} \Rightarrow \frac{3}{4}+\frac{n}{4}=\frac{2+m}{4} \Rightarrow \boxed{n=1}$$

$$\Rightarrow m+n=2+1=\boxed{3}$$

۴-

$$(3g-f)(x)=\frac{9}{3+\sin x}-\frac{\cancel{(3-\sin)}(9+\sin^2+3\sin)}{\cancel{(3-\sin)}(3+\sin)}$$

$$\Rightarrow \frac{9-9-\sin^2 x-3\sin x}{3+\sin x} \Rightarrow \frac{-\sin x\,\cancel{(3+\sin x)}}{\cancel{3+\sin x}}$$

$$=-\sin x \Rightarrow (3g-f)'(x)=-\cos x$$

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$$\Rightarrow -\cos\left(\frac{5\pi}{3}\right)=\boxed{-\frac{1}{2}}$$

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$$(f\circ g)'(\sqrt[5]{3})$$

$f\circ g(x)=-x \rightarrow$

$(f\circ g)'(x)=-1 \rightarrow (f\circ g)'(\sqrt[5]{3})=-1$

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$$f(g(x))=f\left(\frac{+1}{2x^5}\right) \Rightarrow \frac{-1}{\sqrt[5]{\frac{1}{x^5}}}=x$$

$$\Rightarrow (f\circ g)'(x)=(x)'=\boxed{1}$$

۶-

$$\frac{1+\sin^2 x-2\sin x}{1+\sin^2 x+2\sin x}=xg(x)+1$$

$$\Rightarrow g(x)=\frac{-4\sin x}{(x)(\sin x+1)^2} \Rightarrow \lim_{x\to 0} g(x) \xrightarrow{\frac{0}{0}}$$

$$\Rightarrow \text{Hop}=\frac{-4\cos x}{(1)(\sin x+1)^2-(x)(2(\sin x+1)\cos x)}$$

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$$\Rightarrow \frac{-4\cos(0)}{(\sin(0)+1)^2}=\boxed{-4}$$

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قرینه‌سازی $\Rightarrow f(x)=-(x^2+1) \Rightarrow f'(x)=-2x$

$$f'(\alpha)f(\alpha)=-1 \Rightarrow (-2\alpha)(2\alpha)=-1 \Rightarrow \alpha^2=\frac{1}{4}$$

$$\xrightarrow{\alpha>0} \alpha=\frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right)=-\left(\left(\frac{1}{2}\right)^2+1\right)=-\frac{5}{4}$$

$\Rightarrow \left(+\frac{1}{2},-\frac{5}{4}\right) \Rightarrow y=-\frac{5}{4} \Rightarrow$ $\boxed{\text{فاصله از مبدأ} = \frac{5}{4}}$

خط که تابع ثابته

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$$f(x)=8x^2\sqrt{x}+6\sqrt{x} \rightarrow f'(x)=20x\sqrt{x}+\frac{3}{\sqrt{x}}$$

نقطه $(\alpha, a\alpha)$ را محل تماس می‌گیریم $\Rightarrow$

$$f'(\alpha)=20\alpha\sqrt{\alpha}+\frac{3}{\sqrt{\alpha}}=a \Rightarrow 20\alpha^2\sqrt{\alpha}+3\sqrt{\alpha}=a\alpha$$

$$\Rightarrow a\alpha=20\alpha^2\sqrt{\alpha}+3\sqrt{\alpha}=8\alpha^2\sqrt{\alpha}+6\sqrt{\alpha}$$

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$$\Rightarrow 12\alpha^2\cancel{\sqrt{\alpha}}=3\cancel{\sqrt{\alpha}} \Rightarrow \alpha^2=\frac{1}{4} \Rightarrow \alpha=\pm\frac{1}{2}$$

غ ق ق

$$f'\left(\frac{1}{2}\right)=8\left(\frac{1}{2}\right)^2\sqrt{\frac{1}{2}}+6\sqrt{\frac{1}{2}}=\frac{a}{2} \Rightarrow \frac{8}{\sqrt{2}}=\frac{a}{2}$$

$$\Rightarrow a=\boxed{8\sqrt{2}}$$

$(fog)'(x) = g'(x)\, f'(g(x))$ ۱ -۱۰

$g(x) = (x^۲-۱)^{-۱/۲} \xrightarrow{مشتق} \frac{-۱}{۲}(x^۲-۱)\,۲x = -x(x^۲-۱)$

$f(x) = (\underbrace{x(x-۱)}_{x^۲-x})^۳ \xrightarrow{مشتق} ۳(x^۲-x)(۲x-۱)$

$\rightsquigarrow g'(\frac{\sqrt{۵}}{۲}) = -(\frac{\sqrt{۵}}{۲})(\frac{۱}{۴}) = \frac{-\sqrt{۵}}{۸}$

$f'(g(x)) = ۳(\frac{۱}{x^۲-۱} - \frac{۱}{\sqrt{x^۲-۱}})(\frac{۲}{\sqrt{x^۲-۱}} - ۱)$

$\Rightarrow (۳)(\frac{۱}{\frac{۱}{۴}} - \frac{۱}{\frac{۱}{۲}})(\frac{۲}{\frac{۱}{۲}} - ۱)$

$\Rightarrow (۳)(۲)(۳) = ۱۸$

$\rightsquigarrow (fog)'(x) = \cancel{۱۸}^{۹} \times (\frac{-\sqrt{۵}}{\cancel{۸}_{۴}}) = \frac{-۹\sqrt{۵}}{۴}$

$\rightsquigarrow \frac{+\cancel{۹}^{۳}\sqrt{۵}}{۴} \times \frac{۱}{+\cancel{۴۸}_{۱۶}\sqrt{۵}} = \boxed{\frac{۳\sqrt{۵}}{۳۲۰}}$

۶۴

$d: y = cx$ -۹

$f(\alpha) = \frac{\sqrt{\alpha}}{-۲\alpha^۲+\alpha+۱} = c\alpha$

$\Rightarrow \frac{۱}{-۲\alpha^۲+\alpha+۱} = c\sqrt{\alpha} \rightsquigarrow c\sqrt{\alpha}(-۲\alpha^۲+\alpha+۱) = ۱$

$\Rightarrow -۲c\alpha^۲\sqrt{\alpha} + c\sqrt{\alpha}\,\alpha + c\sqrt{\alpha} = ۱$

$\Rightarrow -۵\alpha\sqrt{\alpha} + \frac{۳}{۲}\sqrt{\alpha} + \frac{۱}{۲\sqrt{\alpha}} = ۰$

$\Rightarrow -۱۰\alpha^۲ + ۳\alpha + ۱ = ۰ \rightarrow \alpha = -۱/۵ , \boxed{۱/۲}$

$f(\alpha) = \frac{\sqrt{۱/۲}}{(-۲)(\frac{۱}{۲})^۲ + (\frac{۱}{۲}) + ۱} = \boxed{\frac{\sqrt{۲}}{۲}}$ ۲

۱۰

$(fog(\frac{\sqrt{۵}}{۲}))' = g'(\frac{\sqrt{۵}}{۲})^- \times f'(g(\frac{\sqrt{۵}}{۲})^-)$

$g(x) = (x^۲-۱)^{-\frac{۱}{۲}} \rightarrow g'(x) = \frac{-۱}{۲}(x^۲-۱)^{-\frac{۳}{۲}} \times ۲x \rightarrow g'(\frac{\sqrt{۵}}{۲}^-) = \frac{۱}{\sqrt{(\frac{۵}{۴})^- - ۱}} = \frac{۱}{\sqrt{(\frac{۱}{۴})^-}} = \frac{۱}{(\frac{۱}{۲})^-} = ۲^+$

$f'(۲^+) = ((۲x)^۳)' = (۸x^۳)' = ۲۴x^۲ = ۲۴ \times ۴$

$\rightarrow g'(\frac{\sqrt{۵}}{۲}^-) \times f'(g(\frac{\sqrt{۵}}{۲})^-) = -۴\sqrt{۵} \times ۴ \times ۲۴ \longrightarrow \frac{۲۴ \times ۴ \times (-۴\sqrt{۵})}{-۴۸\sqrt{۵}} = ۸$