۱۴٫۵

نام و نام خانوادگی: کوروش شیرانی فر — پاسخنامه تشریحی تکلیف شماره ۲۴ — کلاس دوازدهم A

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شیب منحنی در نقطه ۳ = شیب خط

$$\text{شیب خط} = \frac{y_2-y_1}{x_2-x_1} = \frac{5-1}{3-0} = \frac{4}{3} \Rightarrow f'(3) = \frac{4}{3}$$

(۲)

۲

$$\text{شیب خط} = \frac{2-1}{2-(-1)} = \frac{1}{3} \qquad y-2 = \frac{1}{3}(x-2) \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$$

$$f'(x) = \frac{a}{2\sqrt{ax-1}} \xrightarrow[f(x_A)=y_A]{f'(x_A)=\frac{1}{3}} \frac{a}{2y_A} = \frac{1}{3} \Rightarrow a = \frac{2y_A}{3} = \frac{2(x_A+4)}{9}$$

$$\left.\begin{array}{l} y_A = \frac{x_A+4}{3} \\ y_A = \sqrt{ax_A-1} \end{array}\right\} \sqrt{ax_A-1} = \frac{x_A+4}{3} \Rightarrow ax_A-1 = \frac{(x_A+4)^2}{9}$$

(جواب کامل پایین صفحه)

(۱)

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$$4y-3x=n \Rightarrow y = \frac{3}{4}x + \frac{n}{4} \to m = \frac{3}{4} \qquad y' = \frac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2}$$

$$\text{At } x=1:\; y'(1) = \frac{(2+m)(4)-(1+m+1)}{4^2} = \frac{6+3m}{16} = \frac{3}{4} \Rightarrow 3m+6=12 \to m=2$$

$$y(1) = \frac{1^2+2(1)+1}{1+3} = \frac{4}{4} = 1 \qquad \xrightarrow{(1,1)} 4(1)-3(1)=n \to n=1$$

$$m+n=3$$

(۲)

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$$f(x) = \frac{(3-\sin x)(9+3\sin x+\sin^2 x)}{(3-\sin x)(3+\sin x)} = \frac{9+3\sin x+\sin^2 x}{3+\sin x} = \sin x + \frac{9}{3+\sin x}$$

$$g(x) = \frac{3}{3+\sin x} \Rightarrow f(x) = \sin x + 3g(x) \;/\; f'(x) = \cos x + 3g'(x) \Rightarrow 3g'(x) - f'(x) = -\cos x$$

$$-\cos\left(\frac{5\pi}{3}\right) = -\cos(300^\circ) = -\left(\frac{1}{2}\right) = -0.5$$

(۲)

۵

$$① \; x>0 \to f(x) = -\frac{1}{\sqrt[5]{2x}} = -(2x)^{-1/5} \qquad g(x) = \frac{1}{2x^5} = \frac{1}{2}x^{-5}$$

$$f(g(x)) = f\left(\frac{1}{2x^5}\right) = -\frac{1}{\sqrt[5]{2\left(\frac{1}{2x^5}\right)}} = -\frac{1}{\sqrt[5]{\frac{1}{x^5}}} = -\frac{1}{\frac{1}{x}} = -x$$

$$\boxed{(f\circ g)'(\sqrt[5]{3}) = -1}$$

(۲)

$g(x) = \frac{f(x)-1}{x}$ $\quad f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x}$ $\quad \boxed{\lim_{x\to 0} g(x) = -4}$

$f(0) = \left(\frac{-1+0}{1+0}\right)^2 = (-1)^2 = 1 \Rightarrow \lim_{x\to 0} \frac{f(x)-1}{x} = f'(0)$

$u = \frac{-1+\sin x}{1+\sin x} \Rightarrow f(x) = u^2 \Rightarrow f'(x) = 2u \cdot u' \Rightarrow u' = \frac{(\cos x)(1+\sin x) - (\cos x)(-1+\sin x)}{(1+\sin x)^2}$

$= \frac{\cos x + \sin x \cdot \cos x + \cos x - \sin x \cdot \cos x}{(1+\sin x)^2} \xrightarrow{x=0} u(0) = -1 \Rightarrow u'(0) = \frac{2(1)}{(1+0)^2} = 2 \to f'(0) = 2(-1)\times 2 = -4$ ۶

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$-x^2 - 1 = k \Rightarrow x^2 = -k-1 \Rightarrow x_1 = \sqrt{-k-1}$ و $x_2 = -\sqrt{-k-1}$

$y' = -2x \to m_1 = -2x_1$ و $m_2 = -2x_2 \Rightarrow m_1 \cdot m_2 = -1$

$(-2x_1)(-2x_2) = -1 \Rightarrow 4(x_1 x_2) = -1 \Rightarrow 4(-\sqrt{-k-1} \cdot \sqrt{-k-1}) = -1$ ۷

$\Rightarrow 4(k+1) = -1 \Rightarrow k = -1/25 \to$ فاصله $y=0$ تا $y=-1/25$ $= |-1/25| = 1/25 = \boxed{\frac{5}{4}}$

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$f(x) = 4x^{2/5} + 3x^{0/5} \xrightarrow[(0,0)]{(a, f(a))} m = \frac{f(a)-0}{a-0} = \frac{f(a)}{a}$ $\quad m = \frac{4\cdot(\frac{1}{4})+3}{\sqrt{\frac{1}{2}}} = 4\sqrt{2}$

$f'(x) = 10x^{1/5} + 1/5x^{-0/5}$

a [illegible] $\to \frac{4a^{2/5} + 3a^{0/5}}{a} = 10a^{1/5} + 1/5a^{-0/5}$ ۸

$\Rightarrow 4a^{1/5} + 3a^{-0/5} = 10a^{1/5} + 1/5a^{-0/5} \to 1/5a = 6a^{1/5}$

$1/5a^{-0/5} : 1 = 4a^2 \Rightarrow a^2 = \frac{1}{4} \Rightarrow a = 0/5 \to m = f'(0/5) = 10(0/5)^{1/5} + 1/5(0/5)^{-0/5}$

$\to m = 10\left(\frac{1}{2\sqrt{2}}\right) = \boxed{4\sqrt{2}}$

۱/۵

نقطه $\to y = ax$ $\quad A(\alpha, a\alpha)$

$f(\alpha) = \frac{\sqrt{\alpha}}{-2\alpha^2+\alpha+1} = a\alpha \to a\sqrt{\alpha}(-2\alpha^2+\alpha+1) = 1 \to -2a\alpha^{\frac{5}{2}} + a\alpha^{\frac{1}{2}} + a\alpha^{\frac{3}{2}} = 1$

$\xrightarrow{\text{مشتق}} -5a\alpha^{\frac{3}{2}} + \frac{3}{2}a\alpha^{\frac{1}{2}} + \frac{1}{2}a\alpha^{-\frac{1}{2}} = 0 \xrightarrow[\times 2\sqrt{\alpha}]{\div a} -10\alpha^2 + 3\alpha + 1 = 0 \to \begin{cases} \alpha = \frac{-1}{5} \\ \alpha = \frac{1}{2} \end{cases}$ ۹

$f(\alpha) = \frac{\sqrt{\frac{1}{2}}}{-2(\frac{1}{2})^2 + \frac{1}{2} + 1} = \frac{\sqrt{2}}{2}$

۵

$(fog(\frac{\sqrt{5}}{2}))' = g'(\frac{\sqrt{5}}{2})^- \times f'(g(\frac{\sqrt{5}}{2})^-)$

$g(x) = (x^2-1)^{-\frac{1}{2}} \to g'(x) = \frac{-1}{2}(x^2-1)^{-\frac{3}{2}} \times 2x \to g'(\frac{\sqrt{5}}{2}^-) = \frac{1}{\sqrt{(\frac{5}{4})^- - 1}} = \frac{1}{\sqrt{(\frac{1}{4})^-}} = \frac{1}{(\frac{1}{2})^-} = 2^+$

$f'(2^+) = ((2x)^3)' = (8x^3)' = 24x^2 = 24 \times 4$ ۱۰

$\to g'(\frac{\sqrt{5}}{2}^-) \times f'(g(\frac{\sqrt{5}}{2})^-) = -4\sqrt{5} \times 4 \times 24 \to \frac{24 \times 4 \times (-4\sqrt{5})}{-48\sqrt{5}} = 8$

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سؤال ۲

$m = \frac{2-1}{2-(-1)} = \frac{1}{3}$

$f(x) = \sqrt{ax-1} \xrightarrow[\text{خط مماس}]{\text{شیب}} f'(x) = \frac{a}{2\sqrt{ax-1}} \rightarrow \frac{a}{2\sqrt{ax-1}} = \frac{1}{3} \rightarrow 3a = 2\sqrt{ax-1} \quad (I)$

معادله خط گذرنده از ۲ نقطه $= y = \frac{1}{3}x + \frac{4}{3} \rightarrow 3y = x+4 \rightarrow x+4 = 3\sqrt{ax-1} \quad (II)$

$\xrightarrow{I \text{ و } II} x+4 = \left(\frac{3a}{2}\right)3 = \frac{9a}{2} \rightarrow x = 4.5a - 4$

$\xrightarrow{II} 4.5a - 4 + 4 = 3\sqrt{a(4.5a-4)-1} \rightarrow 9a^2 - 16a - 4 = 0 \rightarrow \begin{cases} a = 2 \\ a = -\frac{2}{9} \end{cases}$

$f(5) = \sqrt{2(5)-1} = \sqrt{9} = 3$