نام و نام خانوادگی کوروش شیرانی فر پاسخنامه تشریحی تکلیف شماره ۲۴ کلاس دوازدهم A

۱

شیب منحنی در نقطه ۳ = شیب خط

$$\text{شیب خط} = \frac{y_2-y_1}{x_2-x_1} = \frac{5-1}{3-0} = \frac{4}{3} \Rightarrow f'(3) = \frac{4}{3}$$

۲

$$\text{شیب خط} = \frac{2-1}{2-(-1)} = \frac{1}{3} \qquad y-2 = \frac{1}{3}(x-2) \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$$

$$f'(x) = \frac{a}{2\sqrt{ax-1}} \xrightarrow[f(x_A)=y_A]{f'(x_A)=\frac{1}{3}} \frac{a}{2y_A} = \frac{1}{3} \Rightarrow a = \frac{2y_A}{3} = \frac{2(x_A+4)}{9}$$

$$\left.\begin{array}{l} y_A = \frac{x_A+4}{3} \\ y_A = \sqrt{ax_A-1} \end{array}\right\} \sqrt{ax_A-1} = \frac{x_A+4}{3} \Rightarrow ax_A-1 = \frac{(x_A+4)^2}{9}$$

۳

$$4y-3x=n \Rightarrow y = \frac{3}{4}x + \frac{n}{4} \rightarrow m = \frac{3}{4} \qquad y' = \frac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2}$$

$$\text{At } x=1:\ y'(1) = \frac{(2+m)(4)-(1+m+1)}{4^2} = \frac{6+3m}{16} = \frac{3}{4} \Rightarrow 3m+6=12 \rightarrow m=2$$

$$y(1) = \frac{1^2+2(1)+1}{1+3} = \frac{4}{4} = 1 \xrightarrow{(1,1)} 4(1)-3(1) = n \rightarrow n=1$$

$$m+n=3$$

۴

$$f(x) = \frac{(3-\sin x)(9+3\sin x+\sin^2 x)}{(3-\sin x)(3+\sin x)} = \frac{9+3\sin x+\sin^2 x}{3+\sin x} = \sin x + \frac{9}{3+\sin x}$$

$$g(x) = \frac{3}{3+\sin x} \Rightarrow f(x) = \sin x + 3g(x) \ /\ f'(x) = \cos x + 3g'(x) \Rightarrow 3g'(x) - f'(x) = -\cos x$$

$$-\cos\left(\frac{5\pi}{3}\right) = -\cos(300^\circ) = -\left(\frac{1}{2}\right) = -0.5$$

۵

$$① \ x>0 \rightarrow f(x) = -\frac{1}{\sqrt[5]{2x}} = -(2x)^{-1/5} \qquad g(x) = \frac{1}{2x^5} = \frac{1}{2}x^{-5}$$

$$f(g(x)) = f\left(\frac{1}{2x^5}\right) = -\frac{1}{\sqrt[5]{2\left(\frac{1}{2x^5}\right)}} = -\frac{1}{\sqrt[5]{\frac{1}{x^5}}} = -\frac{1}{\frac{1}{x}} = -x$$

$$(f\circ g)'_{(\sqrt[5]{3})} = -1$$

$g(x) = \dfrac{f(x)-1}{x}$ $\quad f'(0) = \lim\limits_{x\to 0} \dfrac{f(x)-f(0)}{x}$ $\quad \boxed{\lim\limits_{x\to 0} g(x) = -4}$ ۶

$f(0) = \left(\dfrac{-1+0}{1+0}\right)^2 = (-1)^2 = 1 \Rightarrow \lim\limits_{x\to 0} \dfrac{f(x)-1}{x} = f'(0)$

$u = \dfrac{-1+\sin x}{1+\sin x} \Rightarrow f(x) = u^2 \Rightarrow f'(x) = 2u\cdot u' \Rightarrow u' = \dfrac{(\cos x)(1+\sin x) - (\cos x)(-1+\sin x)}{(1+\sin x)^2}$

$= \dfrac{\cos x + \sin x\cdot\cos x + \cos x - \sin x\cdot\cos x}{(1+\sin x)^2} \xrightarrow{x=0} u(0) = -1 \Rightarrow u'(0) = \dfrac{2(1)}{(1+0)^2} = 2 \to f'(0) = 2(-1)\times 2 = -4$

$-x^2 - 1 = k \Rightarrow x^2 = -k-1 \Rightarrow x_1 = \sqrt{-k-1}$ و $x_2 = -\sqrt{-k-1}$

$y' = -2x \to m_1 = -2x_1$ و $m_2 = -2x_2 \to m_1\cdot m_2 = -1$

$(-2x_1)(-2x_2) = -1 \Rightarrow 4(x_1x_2) = -1 \Rightarrow 4\left(-\sqrt{-k-1}\cdot\sqrt{-k-1}\right) = -1$ ۷

$\Rightarrow 4(k+1) = -1 \Rightarrow k = -1.25 \to$ فاصله از $y=0$ تا $y=-1.25$ $= |-1.25| = 1.25 = \boxed{\dfrac{5}{4}}$

$f(x) = 4x^{2.5} + 3x^{0.5} \quad \xrightarrow[(0,0)]{(a,f(a))} \quad m = \dfrac{f(a)-0}{a-0} = \dfrac{f(a)}{a}$

$f'(x) = 10x^{1.5} + 1.5x^{-0.5}$

در نقطه a $\to \dfrac{4a^{2.5} + 3a^{0.5}}{a} = 10a^{1.5} + 1.5a^{-0.5}$ ۸

$\Rightarrow 4a^{1.5} + 3a^{-0.5} = 10a^{1.5} + 1.5a^{-0.5} \to 1.5a^{-0.5} = 6a^{1.5}$

$1.5a^{-0.5} : 1 = 4a^2 \Rightarrow a^2 = \dfrac{1}{4} \Rightarrow a = 0.5 \to m = f'(0.5) = 10(0.5)^{1.5} + 1.5(0.5)^{-0.5}$

$\to m = 10\left(\dfrac{1}{2\sqrt{2}}\right) = \boxed{4\sqrt{2}}$

۹

۱۰