

خط مماس: $y = mx + b$
 $m = \frac{4}{3} \Rightarrow f'(x) = \frac{4}{3}$
 در نقطه $m = 3$ مماس بر دایره.

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$(2, 2), (-1, 1) \rightarrow m = \frac{2-1}{2-(-1)} = \frac{1}{3} \Rightarrow y = \frac{1}{3}x + \frac{5}{3} = \sqrt{ax-1}$
 $\Rightarrow x + 5 = 3\sqrt{ax-1} \Rightarrow x^2 + 10x + 25 = 9ax - 9 \Rightarrow x^2 + (10-9a)x + 34 = 0$
 $\Delta = 0 \Rightarrow (10-9a)^2 = 4 \cdot 9 \cdot 34 \Rightarrow 10-9a = \pm 6 \Rightarrow a = \frac{4}{9} \rightarrow$ (کسر صحیح)
 $f(x) = -\sqrt{2x-1} = \sqrt{9} = 3 \leftarrow 1-9a = -1 \Rightarrow a = \frac{2}{9}$

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$f(x) = 3x + n \rightarrow y = \frac{3}{\epsilon}x + \frac{n}{\epsilon} \Rightarrow y' = 3 \Rightarrow y_r = \frac{x^2 + mx + 1}{x + 3} \rightarrow (m+2)$
 $y_r = \frac{\epsilon(3x+n) - (m+2)}{\epsilon x \epsilon} \Rightarrow \frac{3(m+2)}{14} = \frac{3}{\epsilon} \Rightarrow m = 2$
 $\frac{x^2 + 2x + 1}{x + 3} \Rightarrow x = 1 \rightarrow y = 1, \Rightarrow 1 = \frac{3}{\epsilon} + \frac{n}{\epsilon} \Rightarrow n = 1, m+n = 3$

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$g(x) = 3(-1)(3 + \sin u)^{-2} \cos u \Rightarrow g'(\frac{5\pi}{6}) = -\frac{3 \cdot \frac{1}{2}}{(3 - \sqrt{3})^2} = \frac{-4}{(4 - \sqrt{3})^2}$
 $f(x) = \frac{(3 - \sin u)(9 + 3\sin u + \sin^2 u)}{(3 - \sin u)(3 + \sin u)} \Rightarrow f(x) = \sin u + \frac{9}{3 + \sin u}$
 $f'(\frac{5\pi}{6}) = \frac{1}{2} \left(1 - \frac{34}{(4 - \sqrt{3})^2} \right) \Rightarrow 3g'(\frac{5\pi}{6}) - f'(\frac{5\pi}{6}) = \left(\frac{-1}{2} \right)$

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$g'(x) \cdot f'(g(x)) \Rightarrow f'(g(x))' = (-1) \leftarrow$
 $g(x) = \frac{1}{2ax} \text{ و } f(x) = \frac{-1}{a\sqrt{2x}} \Rightarrow f(g(x)) = -x$

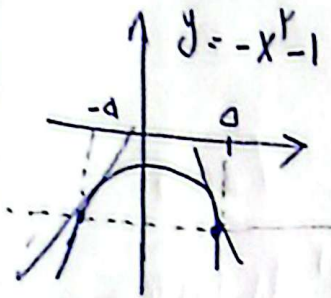
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$$f(x) = u \cdot g(x) + 1 \Rightarrow g(x) = \frac{f(x) - 1}{u}$$

$$\therefore f(x) - 1 = \frac{\sin^2 \pi + \sin^2 \pi - \sin^2 \pi - 1 - \sin^2 \pi}{(1 + \sin^2 \pi)^2} \Rightarrow g(x) = \frac{-\epsilon \sin^2 \pi}{u(1 + \sin^2 \pi)^2}$$

$$\lim_{u \rightarrow 0} g(x) = \frac{-\epsilon \pi}{u(1)} = \textcircled{-\epsilon}$$

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$$y = -x^2 - 1 \rightarrow y' = -2x \begin{cases} \rightarrow a \\ \rightarrow -a \end{cases} \Rightarrow \begin{cases} y'_1 = -2a \\ y'_2 = +2a \end{cases}$$

$$(-2a)(2a) = -4 \Rightarrow a = \frac{1}{\sqrt{\epsilon}}$$

$$y = -\left(\frac{1}{\sqrt{\epsilon}}\right)^2 - 1 = \textcircled{\frac{-1-\epsilon}{\epsilon}}$$

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$$d \rightarrow y = mx \Rightarrow mx = \sqrt{x} (\epsilon \sqrt{x} + 4) \xrightarrow{x \sqrt{x}} m \sqrt{x} = \sqrt{x} (\epsilon \sqrt{x} + 4)$$

$$t = \sqrt{x}, x = t^2 \Rightarrow mt = \sqrt{t^2} (\epsilon \sqrt{t^2} + 4) \Rightarrow mt = \sqrt{t^2} (\epsilon t + 4)$$

$$m = \sqrt{t^2} \left(\frac{\epsilon \sqrt{t^2}}{\sqrt{t^2}} + \frac{4}{\sqrt{t^2}} \right) \leftarrow \begin{cases} \text{تشفير لـ } \epsilon \\ \text{تشفير لـ } 4 \end{cases} \left\{ \begin{array}{l} \epsilon t^2 - m t + 4 = 0 \\ 3 \epsilon t^2 = m \\ \epsilon t^2 - 3 \epsilon t \epsilon = -4 \\ -2 \epsilon t \epsilon = -4 \end{array} \right.$$

$$m = \sqrt{t^2} \left\{ \begin{array}{l} t^2 = \frac{1}{\epsilon} \\ t = \frac{1}{\sqrt{\epsilon}} \end{array} \right.$$

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$$f(g(x)) = \left(g(x) \left[\frac{g(x)}{\sqrt{x}} \right] \right)^{\sqrt{x}} = \sqrt{x} g(x)^{\sqrt{x}} = \sqrt{x} \sqrt{x} g(x)^{\sqrt{x}}$$

$$\left[g\left(\frac{\sqrt{a}}{\sqrt{x}}\right) \right] = \sqrt{x}$$

$$\text{جواب} = \frac{\epsilon \sqrt{x} \sqrt{x}}{-\epsilon \sqrt{x}} = \textcircled{\frac{-\sqrt{a}}{a}}$$

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