متن شناسی ۱۶٫۵

-۱

$m_{AB} \Rightarrow y = mx + 1$, $\Delta = rm + 1$ $\boxed{m = \frac{4}{3}}$, $f'(r) = m_{AB} \longrightarrow \boxed{f'(r) = \frac{4}{3}}$ ✓

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$A\Big|\begin{smallmatrix}2\\1\end{smallmatrix}$, $B\Big|\begin{smallmatrix}-1\\0\end{smallmatrix}$

$m_{AB} = \frac{\Delta y}{\Delta x} \rightarrow \frac{2-1}{2-(-1)} = \frac{1}{3}$

$y = \frac{1}{3}(x+1)$

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$y_{L} = y_f \longrightarrow \frac{A+4}{3} = \sqrt{aA-1} \longrightarrow (A+4)^2 = 9(aA-1) \Rightarrow A^2+8A+16 = 9aA-9$

$\left.\begin{array}{l} A^2+(8-9a)A+25=0 \\ \Delta=0 \end{array}\right\} \Longrightarrow 8-9a = \pm 10 \begin{cases} +10 \rightarrow -\frac{2}{9} \text{ غ ق ق} \\ -10 \rightarrow 2 \text{ ق ق} \end{cases}$, $(A-5)^2 = 0 \Rightarrow A = 5$

$\boxed{f(5) = \sqrt{10-1} = 3}$ ✓

$f'(A) = f'(5) = m_y \Longrightarrow \frac{1}{\sqrt{10-1}} = \frac{1}{3}$

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$f(x) = g(x)$

$f'(x) = g'(x) \longrightarrow \frac{x^2+mx+1}{x+3} \rightarrow \frac{\overset{2x^2+mx+6x+3m}{\cancel{(2x+m)(x+3)}} - (x^2+mx+1)}{(x+3)^2} = \frac{3}{4} \Rightarrow \frac{x^2+6x+3m-1}{(x+3)^2} = \frac{3}{4}$

$g'(x) = \frac{3}{4}$

$4x^2+24x+12m-4 = 3x^2+18x+27$, $x^2+6x+12m-31 = 0 \xrightarrow{\Delta=0}$

$\Delta = 36 - 4(12m-31) = 0 \quad -48m = -160 \quad \boxed{m = \frac{10}{3}}$

$x^2+6x+\frac{12\times 10}{3}-31 = x^2+6x+9 \Rightarrow (x+3)^2 = 0$

$\frac{x^2+\frac{10}{3}x+1}{(x+3)} \longrightarrow \frac{(x+3)(3x+1)}{(x+3)} \begin{cases} -3 \text{ غ ق ق} \\ -\frac{1}{3} \text{ ق ق} \end{cases}$, $f(-\frac{1}{3}) = 0$, $g(-\frac{1}{3}) = 0$, $\boxed{m+n = \frac{10}{3}+1 = \frac{13}{3}}$

$4y = 3x+n$

$4y = \cancel{3(-\frac{1}{3})}+n$, $\boxed{n = 1}$

$(1, \frac{2+m}{4}) \rightarrow y' = \frac{(2+m)4-(2+m)}{16} = \frac{3(2+m)}{16} = \frac{3}{4} \rightarrow m = 2$

$m+n = 2+1 = 3$

۴- (۱)

$3g'(\frac{5\pi}{3}) - f'(\frac{5\pi}{3}) = (3g(x) - f(x))'(\frac{5\pi}{3})$

$\rightarrow (3g-f)(x) = \left(\frac{9}{3+\sin x} - \frac{27-\sin^3 x}{9-\sin^2 x}\right) = \frac{9}{3+\sin x} - \frac{(3-\sin x)(9+\sin^2 x+3\sin x)}{(3-\sin x)(3+\sin x)} = -\sin x$

$\rightarrow (3g-f)'(x) = -\cos x \rightarrow (3g-f)'(\frac{5\pi}{3}) = -\cos(\frac{5\pi}{3}) = \frac{-1}{2}$

۵- (۱,۵)

$g'(\sqrt[5]{2})\, f'(g(\sqrt[5]{2})) = (f\circ g)'(\sqrt[5]{2})$

$f\circ g(x) = x \quad \boxed{f\circ g(x)' = 1}$ $\qquad f\circ g(x) = \frac{-1}{\sqrt[5]{2(\frac{1}{2x^5})}}$

$f\circ g = \frac{1}{\sqrt[5]{\frac{1}{x^5+|x^5|} + \left|\frac{1}{x^5+|x^5|}\right|}} = \frac{1}{\sqrt[5]{\frac{1}{2x^5}+\frac{1}{2x^5}}} = \frac{1}{\sqrt[5]{\frac{1}{x^5}}} = \frac{1}{1/x} = x$

$\rightarrow f\circ g(x) = -x \rightarrow (f\circ g)'(x) = -1 \rightarrow (f\circ g)'(\sqrt[5]{2}) = -1$

۶-

$g(x) = \frac{f(x)-1}{x} \longrightarrow \frac{\frac{(-1+\sin x)^2}{(1+\sin x)^2}-1}{x} = \frac{(-1+\sin x)^2-(1+\sin x)^2}{x(1+\sin x)^2} = \frac{1+\sin^2 x-2\sin x-1-\sin^2 x-2\sin x}{x(1+\sin x)^2}$

$\frac{-4\sin x}{x(1+\sin x)^2} = g(x) \quad \lim_{x\to 0}\frac{-4\sin x}{x(1+\sin x)^2} = \frac{0}{0} \xrightarrow{\text{هم‌ارزی}} \frac{-4\cancel{x}}{\cancel{x}(1+x)^2} = \boxed{-4}$ (۲)

۷-

$g(x) = -f(x) \longrightarrow g(x) = -x^2-1 \qquad d \parallel x \longrightarrow m_d = 0 \Longrightarrow y_\alpha = y_\beta$

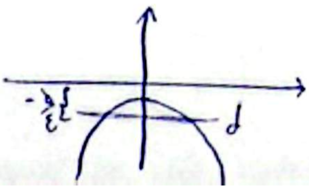
$g'(x) = -2x$

$\alpha, \beta \qquad -2\alpha = \frac{-1}{-2\beta} \Rightarrow -4\alpha\beta = 1 \qquad g(\alpha) = g(\beta) \rightarrow -\alpha^2-1 = -\beta^2-1$

$\alpha^2-\beta^2 = 0$

$(\alpha-\beta)(\alpha+\beta) = 0 \quad \boxed{\alpha = -\beta}$ (۲)

$-4(-\beta)(\beta) = 1 \qquad 4\beta^2 = 1 \qquad \boxed{\beta = 1/2,\ \alpha = -1/2} \qquad \boxed{g(\beta) = -1/4 - 1 = -\frac{5}{4}}$



فاصله d از مبدا مختصات $\frac{5}{4}$ است

-۸

$$f'(a)=\frac{f(a)}{m_a}$$

$$f(x)=2\sqrt{x}\,(4x^2+3) \quad 2\left(\frac{1}{2\sqrt{x}}(4x^2+3)+8x\sqrt{x}\right) = \cancel{\frac{8x^2+6}{2\sqrt{x}}+8x\sqrt{x} = \frac{8x^2+6+16x^2}{2\sqrt{x}}}$$

$$\cancel{2}\times\frac{4x^2+3+16x^2}{\cancel{2}\sqrt{x}}$$

$$\frac{4x^2+3+8x^2}{\sqrt{x}}=\frac{12x^2+3}{\sqrt{x}}$$

$$\cancel{\frac{2\sqrt{x}(4x^2+3)}{x}=\frac{12x^2+3}{\sqrt{x}} \Rightarrow 2x(4x^2+3)=x(12x^2+3) \quad 8x^2+6=12x^2+3}$$

$$\cancel{4x^2=3}$$

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$$\frac{2\sqrt{x}(4x^2+3)}{x}=\frac{4x^2+3+16x^2}{2\sqrt{x}} \Rightarrow 4x(4x^2+3)=x(20x^2+3)$$

$$16x^2+12=20x^2+3 \qquad 12x^2=3 \qquad x^2=\frac{1}{4}$$

$\rightarrow x=\frac{1}{2}$ ق ق

$\rightarrow x=-\frac{1}{2}$ غ ق ق ع

$$m_f=\frac{f(a)}{m_a} \Rightarrow \frac{2\sqrt{\frac{1}{2}}\left(4\left(\frac{1}{2}\right)^2+3\right)}{\frac{1}{2}}=\frac{8\sqrt{\frac{1}{2}}}{\frac{1}{2}}=\frac{16\sqrt{2}}{2}=\boxed{8\sqrt{2}}$$

۹

$$f'(x_0) = \frac{f(x_0)}{x_0}$$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}(-2x^2+x+1) - (-4x+1)(\sqrt{x})}{(-2x^2+x+1)^2} = \frac{\frac{-2x^2+x+1 \cancel{-(-4x+1)(2x)}}{2\sqrt{x}}}{(-2x^2+x+1)^2}$$

$+8x^2-2x$

$$\frac{6x^2-x+1}{2\cancel{\sqrt{x}}(-2x^2+x+1)^{\cancel{2}}} = \frac{\cancel{\sqrt{x}}}{x\cancel{(-2x^2+x+1)}}$$

$4x^2 - x + 1 = -4x^2 + 2x + 2$

$10x^2 - 3x - 1 = 0$ → $x = \frac{1}{2}$ ق ق ; → $x = -\frac{1}{5}$ غ ق ق

$$\frac{\frac{\sqrt{x}}{-2x^2+x+1}}{\frac{x}{1}} = \frac{\sqrt{x}}{x(-2x^2+x+1)}$$

$$f\left(\tfrac{1}{2}\right) = \frac{\sqrt{\frac{1}{2}}}{-2\left(\frac{1}{2}\right)^2+\frac{1}{2}+1} = \frac{\frac{\sqrt{2}}{2}}{1} \qquad \boxed{f\left(\tfrac{1}{2}\right) = \frac{\sqrt{2}}{2}}$$

$\cancel{-2\left(\frac{1}{4}\right)} + \frac{3}{2}$

$-\frac{1}{2} + \frac{3}{2} = 1$

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$$(f\circ g)' = f(g(x))' = g'(x)\, f'(g(x)) \longrightarrow -4\sqrt{5} \times 6(2x)^2 = \boxed{-24\sqrt{5} \times 16}$$

$$g'(x) = \frac{-x}{\sqrt{(x^2-1)^3}} \xrightarrow{x = \frac{\sqrt{5}}{2}} \frac{\frac{\sqrt{5}}{2}}{\sqrt{\left(\frac{1}{4}\right)^3}} = -4\sqrt{5}$$

$$\boxed{\frac{-24\sqrt{5}\times 16}{-48\sqrt{5}} = 8}$$

$\frac{1}{\sqrt{x^2-1}} > \frac{1}{2}$ چون نقطه بحرانی است

$\left[\frac{1}{2}\right] > 2$

$$g\left(\tfrac{\sqrt{5}}{2}\right) = \frac{1}{\sqrt{\frac{1}{4}}} = 2^{+} \rightsquigarrow [2^{+}] = 2$$

$f(2) = (2x)^3$

$f'(2) = 3(2x)^2(2) = 6(2x)^2$

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