۱۸٫۲۵

پارسا فولادی

ریاضی ۳ — A — ۱۴۰۲

۱- $m_{AB} = \frac{5-1}{3-0} = \frac{4}{3} \Rightarrow f'(3) = \frac{4}{3}$ ✓ (۲)

۲- $A(-1,1),\ B(2,2) \Rightarrow m_{AB} = \frac{2-1}{2+1} = \frac{1}{3} \Rightarrow y-2 = \frac{1}{3}(x-2) \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$

$\Rightarrow \sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3} \Rightarrow 3\sqrt{ax-1} = x+4$

$\Rightarrow x^2 + (8-9a)x + 25 = 0 \Rightarrow (8-9a)^2 - 100 = 0 \Rightarrow (8-9a) = \pm 10$ (۲)

$\Rightarrow 8-9a = 10 \Rightarrow a = -\frac{2}{9}$ غ ق ق $\quad 8-9a = -10 \Rightarrow a = 2$

$\Rightarrow f(x) = \sqrt{2x-1} \Rightarrow f(5) = \sqrt{9} = 3$ ✓

۳- $y = \frac{x^2+mx+1}{x+2} \Rightarrow y' = \frac{(2x+m)(x+2) - (x^2+mx+1)}{(x+2)^2} \Rightarrow y'(1) = \frac{3}{4}$

$\Rightarrow 4y - 3x = n \Rightarrow y = \frac{3}{4}x + \frac{n}{4}$ ↑ این باید تقسیم شود (۲)

$\Rightarrow 4(m+2) - 2 - m = 18 \Rightarrow 3m = 6 \Rightarrow m = 2 \Rightarrow \frac{3}{4}x + \frac{n}{4} = y$

$\Rightarrow \frac{3}{4} + \frac{n}{4} = 1 \Rightarrow \frac{3+n}{4} = \frac{4}{4} \Rightarrow n = 1 \Rightarrow 1+3 = 4$ ✓

۴- $g(x) = \frac{3}{3+\sin x} \Rightarrow g'(x) = \frac{-3\cos x}{(3+\sin x)^2} \Rightarrow g'(\frac{5\pi}{3}) = \frac{-3}{(6-\sqrt{3})^2}$ (۱)

$f(x) = \frac{27-\sin^3 x}{9-\sin^2 x} = \frac{(3-\sin x)(9+\sin^2 x+3\sin x)}{(3+\sin x)(3-\sin x)} = \frac{\sin^2 x+3\sin x+9}{3+\sin x}$

$\Rightarrow f'(x) = \frac{(2\sin x\cos x + 3\cos x)(3+\sin x) - (\cos x)(\sin^2 x + 3\sin x + 9)}{(3+\sin x)^2}$

$\Rightarrow f'(\frac{5\pi}{3}) = \frac{\frac{33}{4} + 6\sqrt{3}}{(6-\sqrt{3})^2} \Rightarrow 3g'(\frac{5\pi}{3}) - f'(\frac{5\pi}{3}) = \frac{1+2\sqrt{3}}{33}$

$3g'(\frac{5\pi}{3}) - f'(\frac{5\pi}{3}) = (3g(x) - f(x))'(\frac{5\pi}{3})$

$\rightarrow (3g-f)(x) = \left(\frac{9}{3+\sin x} - \frac{27-\sin^3 x}{9-\sin^2 x}\right) = \frac{9}{3+\sin x} - \frac{(3-\sin x)(9+\sin^2 x+3\sin x)}{(3-\sin x)(3+\sin x)} = -\sin x$

$\rightarrow (3g-f)'(x) = -\cos x \rightarrow (3g-f)'(\frac{5\pi}{3}) = -\cos(\frac{5\pi}{3}) = \frac{-1}{2}$

(۱٫۵ نمره)

۵- $g'(x) \times f'(g(x)) = (f(g(x)))' \Rightarrow x>0 \Rightarrow g(x) = \frac{1}{x^5}$, $f(x) =$

$\Rightarrow f(g(x)) = f(\frac{1}{x^5}) = -x \Rightarrow (fog)'(x) = -1 \to (fog)'(\sqrt[5]{3}) = -1$ $\quad -\frac{1}{5\sqrt[5]{x}}$

۶- $g(x) = \frac{f(x)-1}{x} = \frac{\left(\frac{-1+\sin x}{1+\sin x}\right)^2 - 1}{x} = \frac{-4\sin x}{x(1+\sin x)^2}$ (۲)

هم‌ارزی در همسایگی صفر $\Rightarrow \sin x \sim x \Rightarrow \frac{-4}{x(1+x^2+2x)} = -\frac{4}{1} = -4$ ✓

۷- $f'(\alpha) f'(-\alpha) = -1 \qquad f(x) = -x^2+1 \Rightarrow f'(x) = -2x$

$\Rightarrow (-2\alpha)(2\alpha) = -1 \Rightarrow \alpha^2 = \frac{1}{4}$, $f(\alpha) = f(-\alpha) = -\alpha^2+1 = \frac{5}{4}$ (۲)

عرض نقطه برخورد خط‌ها با هم ← $(0, \frac{5}{4})$ ✓

۸- $f(x) = 2\sqrt{x}(4x^2+c) = mx \Rightarrow 8x^{\frac{5}{2}} + 2cx^{\frac{1}{2}} = mx$

$\Rightarrow f'(x) = 20x^{\frac{3}{2}} + cx^{-\frac{1}{2}} = m \Rightarrow 8x^{\frac{5}{2}} + 2cx^{\frac{1}{2}} = (20\sqrt{x^3} + \frac{c}{\sqrt{x}})x$

$\Rightarrow 8\sqrt{x^5} + 2c\sqrt{x} = 20\sqrt{x^5} + c\sqrt{x} \Rightarrow 12\sqrt{x^5} - c\sqrt{x} = 0$ (۲)

$\Rightarrow 3\sqrt{x}(4x^2-1) = 0$; $x=0$ غ ق ق ؛ $x = \pm\frac{1}{2} \Rightarrow x = -\frac{1}{2}$ غ ق ق $\Rightarrow x = \frac{1}{2}$ ✓

$\Rightarrow m = 20 \times \sqrt{\frac{1}{8}} + 4 \times \sqrt{2} = 8\sqrt{2}$

۹- $\sqrt{x} = t \Rightarrow f(x) = \frac{t}{-2t^4+t^2+1} \Rightarrow \frac{t}{-2t^4+t^2+1} = at^2$ خط مماس (۱٫۵)

$-2at^5 + at^3 + at - 1 = 0 \xrightarrow{\text{مشتق}} -10at^4 + 3at^2 + a = 0 \Rightarrow a(10t^4 - 3t^2 - 1)$

$\Rightarrow (t^2 - \frac{1}{2})(t^2 + \frac{1}{5}) \Rightarrow t^2 = \frac{1}{2}$ قق ✓ $\quad t^2 = -\frac{1}{5}$ غ ق ق

$\Rightarrow f(\frac{1}{2}) = \quad f(x) = \frac{\sqrt{\frac{1}{2}}}{-2(\frac{1}{2})^2 + \frac{1}{2} + 1} = \frac{\sqrt{2}}{2}$

ب) $f(g(x))' = g'(x) \cdot f'(g(x))$, $g(x) = \frac{1}{\sqrt{x^2-1}} = (x^2-1)^{-\frac{1}{2}}$

$\rightarrow g'(x) = -\frac{1}{2}(x^2-1)^{-\frac{3}{2}} \cdot 2x \rightarrow g'(\frac{\sqrt{5}}{2}) = -4\sqrt{5}$

$x \rightarrow (\frac{\sqrt{5}}{2})^- \rightarrow g(x) = \frac{1}{(\frac{1}{4})^-} = 2^+ \rightarrow$ من اليمين 2^+ (٢)

$\rightarrow x \rightarrow 2^+ \rightarrow [2^+] = 2 \rightarrow f(x) = (2x)^3 = 8x^3 \rightarrow f'(x) = 24x^2 \rightarrow f'(2^+) = 96$

$\rightarrow f'(g(\frac{\sqrt{5}}{2}^-)) \cdot g'(\frac{\sqrt{5}}{2})^- = 96 \times -4\sqrt{5} = 8(-48\sqrt{5})$ ✓

$\rightarrow$ الجواب