۵۲۴ دوازدهم سری A پاسخنامه

۱- $m_{AB} = \frac{5-1}{3-0} = \frac{4}{3} \Rightarrow f'(3) = \frac{4}{3}$

۲- $\left.\begin{array}{l} A(-1,1) \\ B(2,2) \end{array}\right\} \Rightarrow m_{AB} = \frac{2-1}{2+1} = \frac{1}{3} \Rightarrow y-2 = \frac{1}{3}(x-2) \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$

$\Rightarrow \sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3} \Rightarrow 3\sqrt{ax-1} = x+4$

$\Rightarrow x^2 + (8-9a)x + 25 = 0 \Rightarrow (8-9a)^2 - 100 = 0 \Rightarrow (8-9a) = \pm 10$

$\Rightarrow 8-9a = 10 \Rightarrow a = -\frac{2}{9} \Rightarrow$ غ ق ق $\qquad 8-9a = -10 \Rightarrow \boxed{a = 2}$

$\Rightarrow f(x) = \sqrt{2x-1} \Rightarrow f(5) = \sqrt{9} = 3$

۳- $y = \frac{x^2+mx+1}{x+3} \Rightarrow y' = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} \Rightarrow y'(1) = \frac{3}{4}$

$\Rightarrow 4y - 3x = n \Rightarrow y = \frac{3}{4}x + \frac{n}{4}$ نقطه مماس $x_0 = 1$ است

$\Rightarrow 4(m+2) - 2 - m = 12 \Rightarrow 3m = 6 \Rightarrow \boxed{m = 2} \Rightarrow \frac{3}{4}x + \frac{n}{4} = y$

$\Rightarrow \frac{3}{4} + \frac{n}{4} = 1 \Rightarrow \frac{3+n}{4} = \frac{4}{4} \Rightarrow \boxed{n = 1} \Rightarrow \boxed{1+2 = 3}$

۴- $g(x) = \frac{3}{3+\sin x} \Rightarrow g'(x) = \frac{-3\cos x}{(3+\sin x)^2} \Rightarrow g'\left(\frac{5\pi}{4}\right) = \frac{-6}{(6-\sqrt{2})^2}$

$f(x) = \frac{27 - \sin^3 x}{9 - \sin^2 x} = \frac{(3-\sin x)(9+\sin^2 x + 3\sin x)}{(3+\sin x)(3-\sin x)} = \frac{\sin^2 x + 3\sin x + 9}{3+\sin x}$

$\Rightarrow f'(x) = \frac{(2\sin x\cos x + 3\cos x)(3+\sin x) - (\cos x)(\sin^2 x + 3\sin x + 9)}{(3+\sin x)^2}$

$\Rightarrow f'\left(\frac{5\pi}{4}\right) = \frac{\frac{23}{2} + 6\sqrt{2}}{(6-\sqrt{2})^2} \Rightarrow 3g'\left(\frac{5\pi}{4}\right) - f'\left(\frac{5\pi}{4}\right) = \boxed{\frac{1+2\sqrt{2}}{22}}$

۵- $g'(x) f'(g(x)) \to (f(g(x)))' \to x>0 \Rightarrow g(x) = \frac{1}{\cos\theta}$ , $f(x) =$

$\Rightarrow f(g(x)) = f\left(\frac{1}{2\cos\theta}\right) = -x$   $-\frac{1}{\sqrt{2x}}$

۶- $g(x) = \frac{f(x)-1}{x} = \frac{\left(\frac{-1+\sin x}{1+\sin x}\right)^2 - 1}{x} = \frac{-4\sin x}{x(1+\sin x)^2}$

هم‌ارزی در صفر $\sin x \sim x \Rightarrow \frac{-4}{x(1+x^2+2x)} = -\frac{4}{1} = -4$

۷- $f'(\alpha) f'(-\alpha) = -1$    $f(x) = -x^2 - 1 \Rightarrow f'(x) = -2x$

$\Rightarrow (-2\alpha)(2\alpha) = -1 \Rightarrow \alpha^2 = \frac{1}{4}$ , $f(\alpha) = f(-\alpha) = -\alpha^2 - 1 = -\frac{5}{4}$

طول نقطه برابر، عرض نقاط برابر $\left(\pm\frac{1}{2}, -\frac{5}{4}\right)$

۸- $f(x) = 2\sqrt{x}(4x^2 + c) = mx \Rightarrow 8x^{\frac{5}{2}} + cx^{\frac{1}{2}} = mx$

$\Rightarrow f'(x) = 20x^{\frac{3}{2}} + \frac{c}{2}x^{-\frac{1}{2}} = m \Rightarrow 8x^{\frac{5}{2}} + cx^{\frac{1}{2}} = \left(20x^{\frac{3}{2}} + \frac{c}{2\sqrt{x}}\right)x$

$\Rightarrow 12\sqrt{x^5} + \sqrt{x} = 24\sqrt{x^5} + 2\sqrt{x} \Rightarrow 12\sqrt{x^5} - \sqrt{x} = 0$

$\Rightarrow 2\sqrt{x}(4x^2 - 1) = 0$ ، $x = 0$ غ ق ، $x = \pm\frac{1}{2}$ ، $x = -\frac{1}{2}$ غ ق $\Rightarrow x = \frac{1}{2}$

$\Rightarrow m = 20 \times \sqrt{\frac{1}{2^3}} + 4 \times \sqrt{2} = 8\sqrt{2}$

۹- $\sqrt{x} = t \Rightarrow f(x) = \frac{t}{-2t^4 + t^2 + 1} \Rightarrow \frac{t}{-2t^4 + t^2 + 1} = at^2$ مماس (۱)

$-2at^5 + at^3 + at - 1 = 0$   مشتق $-10at^4 + 3at^2 + a = 0 \Rightarrow a(10t^4 - 3t^2 - 1) = 0$

$\Rightarrow \left(t^2 - \frac{1}{2}\right)\left(t^2 + \frac{1}{5}\right) \Rightarrow t^2 = \frac{1}{2}$ ق ق ، $t^2 = -\frac{1}{5}$ غ ق

$\Rightarrow f\left(\frac{1}{2}\right) =$

حل: $f(g(x))' = g'(x) \cdot f'(g(x))$ , $g(x) = \frac{1}{\sqrt{x^2-1}} = (x^2-1)^{-\frac{1}{2}}$

$\Rightarrow g'(x) = -\frac{1}{2}(x^2-1)^{-\frac{3}{2}} \cdot 2x \Rightarrow g'(\frac{\sqrt{5}}{2}) = -4\sqrt{5}$

$x \to (\frac{\sqrt{5}}{2})^- \Rightarrow g(x) = \frac{1}{(\frac{1}{2})^-} = 2^+ \to$ مثبت الاتي $2^+$

$\Rightarrow x \to 2^+ \Rightarrow [2^+] = 2 \Rightarrow f(x) = (2x)^3 = 8x^3 \Rightarrow f'(x) = 24x^2 \Rightarrow f'(2) = 96$

$\Rightarrow f'(g(\frac{\sqrt{5}}{2}^-)) \cdot g'(\frac{\sqrt{5}}{2})^- = 96 \times -4\sqrt{5} =$ ٨ $(-384\sqrt{5})$

$\Rightarrow$ الجواب ٨