SUBJECT
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A گروه دوازدهم ریاضی — تکلیف شماره ۲۴ — آرا آزادی‌نیا

①
$A(0,1)\,,\,B(3,5) \Rightarrow f'(3) = \frac{\Delta y}{\Delta x} = \frac{5-1}{3-0} = \frac{4}{3}$

②
جایگذاری

$f(x) = \sqrt{ax-1}\,,\,A(-1,1)\,,\,B(2,2) \Rightarrow ax+b=y \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$

$\Rightarrow \sqrt{ax-1} = \frac{x+4}{3} \Rightarrow x^2 + 8x + 16 = 9ax - 9 \Rightarrow x^2 + (8-9a)x + 25 = 0$

$\xrightarrow{\Delta = 0 \text{ باید}} (8-9a)^2 - 100 = 0 \Rightarrow (8-9a)^2 = 100 \rightarrow 8 - 9a = 10 \Rightarrow a = -\frac{2}{9}$

$\hookrightarrow 8 - 9a = -10 \Rightarrow a = 2$ ق ق

$f(x) = \sqrt{2x-1} \Rightarrow f(5) = 3$

③
$f(x) = \frac{x^2+mx+1}{x+3} \Rightarrow f'(x) = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2}\,,\, y = \frac{3}{4}x + \frac{n}{4}$

$\Rightarrow f'(1) = \frac{3}{4} = \frac{(2+m)(4) - (2+m)}{16} \Rightarrow 3(2+m) = 12 \Rightarrow m + 2 = 4 \Rightarrow m = 2$

$f(x) = \frac{x^2+2x+1}{x+3} \Rightarrow f(1) = \frac{4}{4} = 1 \Rightarrow 1 = \frac{3}{4} + \frac{n}{4} \Rightarrow \frac{3+n}{4} = 1 \Rightarrow n = 1$ $\Big\}\, m+n = 3$

④
$f(x) = \frac{27 - \sin^3 x}{9 - \sin^2 x} = \frac{(3-\sin x)(\sin^2 x + 3\sin x + 9)}{(\sin x + 3)(3 - \sin x)} = \frac{\sin^2 x + 3\sin x + 9}{3 + \sin x}$

$g(x) = \frac{3}{3+\sin x} \Rightarrow 3g(x) = \frac{9}{3+\sin x} \Rightarrow 3g(x) - f(x) = \frac{9 - \sin^2 x - 3\sin x - 9}{3 + \sin x}$

$= \frac{-(\sin x)(\sin x + 3)}{\sin x + 3} = -\sin x \Rightarrow 3g'(x) - f'(x) = -\cos x \xrightarrow{x = \frac{5\pi}{3}} -\cos\frac{5\pi}{3} = -\frac{1}{2}$

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$$g(x)=\frac{1}{x^5+|x^5|}\xrightarrow{\sqrt[5]{3}>0}\frac{1}{2x^5}\Rightarrow g(\sqrt[5]{3})=\frac{1}{6}$$

$$f(x)=-\frac{1}{\sqrt[5]{x+|x|}}\xrightarrow{\frac{1}{6}>0} f(x)=\frac{-1}{\sqrt[5]{2x}}\Rightarrow f\circ g=\frac{-1}{\sqrt[5]{\frac{2}{2x^5}}}=-x$$

$$\Rightarrow (f\circ g)'(x)=-1$$

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$$f(x)=x\,g(x)+1\Rightarrow g(x)=\frac{f(x)-1}{x}\Rightarrow \lim_{x\to 0} g(x)=c$$ تعریف مشتق در صفر است.

$$f(x)\left(\frac{-1+\sin x}{1+\sin x}\right)^2\Rightarrow f'(x)=2\left(\frac{\cos x(\sin x+1)-\cos x(\sin x-1)}{(1+\sin x)^2}\right)$$

$$\left(\frac{-1+\sin x}{1+\sin x}\right)\Rightarrow f'(0)=2\left(\frac{2\cos(0)}{1+\sin(0)^2}\right)\left(\frac{\sin(0)-1}{1+\sin(0)}\right)=2\times2\times-1=-4$$

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$$f(x)=-x^2-1\Rightarrow f'(x)=-2x\Rightarrow$$

(۸)

$$f(x)=2\sqrt{x}(4x^2+3)\Rightarrow f'(x)=20x\sqrt{x}-\frac{3}{\sqrt{x}}\longrightarrow \frac{\Delta y}{\Delta x}=\frac{f(t)}{t}$$ حد اگر دارد

$$\Rightarrow \frac{2\sqrt{t}(4t^2+3)}{t}=20x^{\frac{3}{2}}+3\sqrt{t}\Rightarrow 8t^{\frac{5}{2}}+6t^{\frac{1}{2}}=20t^{\frac{3}{2}}+3t^{\frac{1}{2}}$$

$$3t^{\frac{1}{2}}=12t^{\frac{5}{2}}\Rightarrow 4t^2=1\Rightarrow t=\frac{1}{2}\Rightarrow f'(\tfrac{1}{2})=20\times\frac{1}{2\sqrt{2}}+3\sqrt{2}=8\sqrt{2}$$

$$2\left(\frac{1}{2\sqrt{x}}\right)\left(4x^2+3\right)+2\sqrt{x}\left(8x^3\right)$$

$$\frac{4x^2+3}{\sqrt{x}}+\frac{16x\sqrt{x}+4\sqrt{x}}{1}=\frac{\qquad}{\sqrt{x}}$$

$$f(x)=\frac{\sqrt{x}}{-2x^2+x+1}\Rightarrow f'(x)=\frac{\left(\frac{1}{2\sqrt{x}}\right)\left(-2x^2+x+1\right)-\left(-4x+1\right)\sqrt{x}}{\left(-2x^2+x+1\right)^2}$$ (9)

$$=\frac{6x^2-x+1}{2\sqrt{x}\left(-2x^2+x+1\right)}=\frac{\sqrt{x}}{2\left(-2x^2+x+1\right)\sqrt{x}}\Rightarrow 6x^2-x+1=-4x^2+2x+2$$

$$\Rightarrow 10x^2-3x-1=0\xrightarrow{\text{دو ریشه}}x=\frac{1}{2}\checkmark \quad x=-\frac{1}{5}\times \Rightarrow f\left(\frac{1}{2}\right)=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

$$f(x)=\left(x[x]\right)^3,\quad g(x)=\frac{1}{\sqrt{x^2-1}},\quad \lim_{x\to\frac{\sqrt{5}}{2}} g(x)=\frac{1}{\frac{1}{2}}=2^+$$ (10)

$$\Rightarrow fog'(\ )=\frac{-24x}{\sqrt{(x^2-1)^5}},\quad fog'\left(\frac{\sqrt{5}}{2}\right)=\frac{-12\sqrt{5}}{\frac{1}{32}}=\frac{-12\sqrt{5}\times 32}{-384\sqrt{5}}$$ (8)