ریحانه مقدم — کلاس دوم ... تکلیف ۲۲ — ۱۹ — A

دوشنبه ۲۳ مهر — 14 Oct. 2024 — ۱۰ ربیع الثانی ۱۴۴۶

(۱)
$A\,|\,\begin{smallmatrix}1\\1\end{smallmatrix}$, $B\,|\,\begin{smallmatrix}3\\5\end{smallmatrix}$ $\Rightarrow m_{AB} = \frac{5-1}{3} = \boxed{\frac{4}{3} = f'(3)}$ — ۲

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(۲) — ۲

$A\,|\,\begin{smallmatrix}-1\\1\end{smallmatrix}$, $B\,|\,\begin{smallmatrix}2\\2\end{smallmatrix}$ $\to m_{AB} = \frac{1}{3}$

$\star\; y = \frac{1}{3}x + \frac{4}{3}$

$\sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3} \to 3\sqrt{ax-1} = x+4$

$x^2 + (8-9a)x + 25 = 0$, $\Delta = 0 \Rightarrow b = 8-9a$, -10

$a = -\frac{2}{9}$ غق ، $a = 2$ ✓

$f(x) = \sqrt{2x-1} \Rightarrow \boxed{f(5) = 3}$

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(۳)

$y = \frac{x^2 + mx + 1}{x+3} \to y' = \frac{(2+m)\times 4 - (1+m+1)}{4^2} = \frac{3}{4}$ شیب

$4(m+2) = 2 - m = 1 \;2 \to \boxed{m = 2}$

$4y - 3x = b$

$y = \frac{x^2 + 2x + 1}{x+3} \xrightarrow{x=1} \frac{1+2+1}{1+3} = 1$ → $A\,|\,\begin{smallmatrix}1\\1\end{smallmatrix}$ — ۲

$1 = \frac{3}{4} + \frac{b}{4} \to \boxed{b = 1}$

$\star\; m + 2 = \boxed{3} \Leftarrow$ جواب

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(۴) — ۱

$g(x) = \frac{3}{3+\sin x} \to g'(x) = \frac{-3\cos x}{(3+\sin x)^2}$, $g'\left(\frac{5\pi}{3}\right) = \frac{-\frac{3}{2}}{\frac{(6-\sqrt{3})^2}{4}} = \frac{-6}{(6-\sqrt{3})^2}$

$f(x) = \frac{27 - \sin^3 x}{9 - \sin^2 x} = \frac{\sin^2 x + 3\sin x + 9}{3 + \sin x} \to f'(x)$ (جواب کامل پایین صفحه)

$x = \frac{5\pi}{3}$: $\frac{-3\sqrt{3}}{2} \cdot \frac{-3\sqrt{3}}{(6-\sqrt{3})^2}$

$\star\; 3g'\left(\frac{5\pi}{3}\right) - f'\left(\frac{5\pi}{3}\right) = \boxed{\frac{1+2\sqrt{3}}{33}}$

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(۵) — ۲

$g'(x) \cdot f'(g(x)) = (f\circ g)'(x)$, $x > 0$

$g(x) = \frac{1}{2x^5}$, $x > 0$

$f(x) = \frac{-1}{\sqrt[5]{2x}}$

$f\left(\frac{1}{2x^5}\right) = -x = f\circ g(x)$ مشتق $\to \boxed{-1}$

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(۶)
$g(x) = \frac{f(x) - 1}{x} = \frac{1 + \sin^2 - 2\sin - 1 - \sin^2 - 2\sin}{(1 + \sin^2 + 2\sin)x}$

$\Rightarrow \lim_{x \to 0} \frac{-4\sin x}{(1 + \sin^2 + 2\sin x)x} \longrightarrow \sin x \sim x$

$\star \longrightarrow \lim_{x \to 0} \frac{-4x}{(1 + x^2 + 2x)x} = -4$

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(۷)
$f(x) = -x^2 - 1 \xrightarrow{f'(x)} -2x \qquad f'(z) \times f'(-z) = -1$

$f(z) = f(-z) = -z^2 - 1 = \frac{-5}{4} \longrightarrow y = -\frac{5}{4}$

$1{,}25 = \left| \frac{-5}{4} \right|$ طول

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(۸)
$f(x) = 2\sqrt{x}(4x^2 + 3) = 8x^{\frac{5}{2}} + 6x^{\frac{1}{2}}$

$f'(x) = 20x^{\frac{3}{2}} + 3x^{-\frac{1}{2}} \longrightarrow 12x^{\frac{5}{2}} - 3x^{\frac{1}{2}} = 0$

$\longrightarrow 3\sqrt{x}(4x^2 - 1) = 0$

$x = 0$ غ ق ق, $x = \frac{1}{2}$

$m = 20 \times \left(\frac{1}{2}\right)^{\frac{3}{2}} + 3 \times \left(\frac{1}{2}\right)^{-\frac{1}{2}} = 8\sqrt{2}$ $\star$

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(۹)
$\sqrt{x} = t$

$\Downarrow f(x) = \frac{t}{-2t^4 + t^2 + 1} = 9t^2 \xrightarrow{\text{مشتق}} -9(10t^4 - 3t^2 - 1) = 0$

$t^2 = -\frac{1}{5}$ غ ق ق
$t^2 = \frac{1}{2}$

$\star\ f\left(\frac{1}{2}\right) = \frac{1}{\sqrt{2}} \div \left[-2\left(\frac{1}{4}\right) + \frac{3}{2}\right] = \frac{1}{\sqrt{2}}$ ⟸ برابر

(۲)

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(۱۰)
$(f \circ g(x))' = g'(x) \times f'(g(x))$

$g(x) = (x^2 - 1)^{-\frac{1}{2}} \longrightarrow g'(x) = -\frac{1}{2}(x^2 - 1)^{-\frac{3}{2}} \times 2x = -4\sqrt{5}$

$x \to 2^+ \to [\ ] \Rightarrow 2 \qquad f(x) = (2x)^3 = 8x^3 \star\star$ ۸ برابر

$f'\left(g\left(\frac{\sqrt{5}}{2}\right)\right) \times g'\left(\frac{\sqrt{5}}{2}\right) \star = 12 \cdot 4\sqrt{5}$ ⟶ $f'(x) = f'(24) = 24x^2$ $\star\star\star$

(۲)

سوال ۱۴

$$3g'\left(\frac{5\pi}{3}\right) - f'\left(\frac{5\pi}{3}\right) = (3g(x) - f(x))'\left(\frac{5\pi}{3}\right)$$

$$\rightarrow (3g - f)(x) = \left(\frac{9}{3+\sin x} - \frac{10 - \sin^2 x}{4 - \sin^2 x}\right) = \frac{9}{3+\sin x} - \frac{(3-\sin x)(9+\sin^2 x + 3\sin x)}{(3-\sin x)(3+\sin x)} = -\sin x$$

$$\rightarrow (3g - f)'(x) = -\cos x \rightarrow (3g - f)'\left(\frac{5\pi}{3}\right) = -\cos\left(\frac{5\pi}{3}\right) = \frac{-1}{2}$$