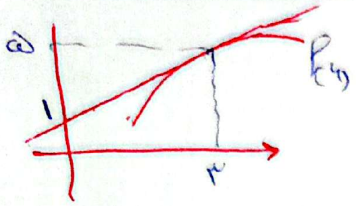
نام و نام خانوادگی ..آیلین پیش‌قدم‌ها‌کارانی.... پاسخنامه تشریحی تکلیف شماره .... کلاس ....دوازدهم‌تجربی....

۱۱٫۵



$f'(3) = ?$

$f(3) = 5$

$y = ax + b$

$3a + 1 = 5 \rightarrow a = \frac{4}{3}$

$y = \frac{4}{3}x + 1 \rightarrow f'(3) = \frac{4}{3}$

(۲) ۱

$f(x) = \sqrt{ax - 1}$

$m_{خط} = \frac{2-1}{2-(-1)} = \frac{1}{3}$ $y = \frac{1}{3}x + \frac{4}{3}$

$f'(x) = \frac{1}{3} = \frac{a}{2\sqrt{ax-1}} \rightarrow \frac{1}{a} = \frac{a^2}{4ax - 4} \rightarrow 4ax - 4 = 9a^2$

$\frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1} \rightarrow x^2 + (8 - 9a)x + 25 = 0 \rightarrow \Delta = 0$ $\rightarrow a = -\frac{4}{9}$ یا $a = 2 \rightarrow f(5) = 3$

(۲) ۲

$f(x) = \frac{x^2 + mx + 1}{x + 3}$ $\rightarrow x = 1 \rightarrow \frac{m+2}{4}$

$y = \frac{3}{4}x + \frac{n}{4} \rightarrow \frac{3}{4} + \frac{n}{4} = \frac{2+m}{4} \rightarrow m - n = 1 \rightarrow n = 1$

$m - n = 1$ ① $\rightarrow$ $m + n = 11$

$y = \frac{3x + n}{4} \rightarrow x = 1 \rightarrow \frac{3+n}{4}$ $y' = \frac{3}{4}$

$m + n = 2 + 1 = 3$

$f'(x) = \frac{x^2 + 6x + 3m - 1}{(x+3)^2} = \frac{3}{4} \xrightarrow{x=1} \frac{3m+6}{16} = \frac{3}{4} \rightarrow m = 2$, $n = 5$

(۱) ۳

$f(x) = \frac{27 - \sin^3 x}{9 - \sin^2 x} = \frac{(3 - \sin x)(\sin^2 x + 3\sin x + 9)}{(3 + \sin x)(3 - \sin x)} = \frac{\sin^2 x + 3\sin x + 9}{\sin x + 3}$

(۰٫۵)

$f'(x) = \frac{\sin^2 x \cos x + 6\sin x\cos x - 9\cos x}{(\sin x + 3)^2} \xrightarrow{x = \frac{5\pi}{3}} \frac{\frac{3}{8} - \frac{3\sqrt{3}}{2} - 9}{(-\frac{\sqrt{3}}{2} + 3)^2}$ ②

(جواب پایین صفحه)

$g(x) = \frac{3}{3 + \sin x} \rightarrow g'(x) = \frac{-3\cos x}{(\sin x + 3)^2} \xrightarrow{x = \frac{5\pi}{3}} \frac{-\frac{3}{2}}{(\frac{-\sqrt{3}}{2} + 3)^2}$ ①

$\frac{-\frac{3}{2} + \frac{3}{8} - \frac{3\sqrt{3}}{2} - 9}{(-\frac{\sqrt{3}}{2} + 3)^2}$

۴

$(f(g(\frac{1}{2})))' = -1$ ✓

$g(x) = \frac{1}{2x^5}$

$f(x) = \frac{-1}{\sqrt[5]{2x}}$

$f(g(x)) = -x$

(۲) ۵

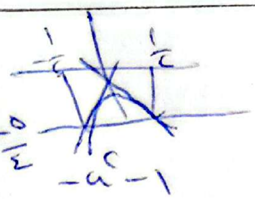
$$g(x) = \frac{\left(\frac{\sin x - 1}{\sin x + 1}\right)^2 - 1}{x}$$

(جواب پایین صفحه) ⓪

$$\lim_{x\to 0} g(x) \xrightarrow{\text{هم ارزی}} \frac{0}{0} \rightarrow \frac{\left(\frac{x-1}{x+1}\right)^2 - 1}{x} \rightarrow \frac{x^2 - 2x + 1 - x^2 - 2x - 1}{(x+1)^2 x} = \lim_{x\to 0} \frac{-4}{(x+1)^2} = \boxed{-4}$$ ۶

$y = ax + b$

$y' = -\frac{1}{a}x + b$



$y = \frac{-5}{4}$

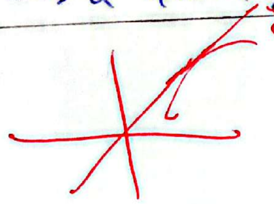
$$y = x - \frac{3}{4} \rightarrow \boxed{y = -\frac{5}{4}}$$ عرض نقطه تلاقی ✓ ②

$x = -x - \frac{3}{4}$ ۷

$$ax + b = -x^2 - 1 \rightarrow x^2 + ax + b + 1 = 0 \rightarrow \Delta = 0 \rightarrow a^2 - 4b - 4 = 0 \rightarrow a^2 = 1 \quad a = \pm 1$$

$$-\frac{1}{a}x + b = -x^2 - 1 \rightarrow x^2 - \frac{1}{a}x + b + 1 \rightarrow \Delta = 0 \rightarrow \frac{1}{a^2} - 4b - 4 = 0 \qquad b = -\frac{3}{4}$$

$$\rightarrow x^2 + x + \frac{1}{4} = 0 \rightarrow 4x^2 + 4x + 1 = 0 \quad \frac{\Delta}{a^2} \quad (2x+1)^2 \rightarrow \boxed{y = -\frac{1}{2}} \quad \boxed{x = -\frac{1}{2}}$$



$y = ax \rightarrow a = ?$

$$h(x) = 8x^2\sqrt{x} + 6\sqrt{x}$$ (جواب پایین صفحه)

$$h'(x) = 20\sqrt{x^3} + \frac{6}{2\sqrt{x}}$$ ✓ ۰/۵ ۸

$$\frac{20x^2 + 3}{\sqrt{x}} = a \rightarrow a\sqrt{x} = 20x^2 + 3$$

$$40x - \frac{a}{2\sqrt{x}} = 0 \rightarrow ax = \sqrt[3]{\left(\frac{a}{80}\right)^2} \qquad 20x^2 - a\sqrt{x} + 3 = 0$$

$$\frac{a}{80} = y \quad \frac{a}{80} \sqrt[3]{y^2} \rightarrow 20\sqrt[3]{y^4} - 80y\sqrt[3]{y^2} + 3 = 0 \rightarrow \frac{a}{80} = \sqrt[4]{\frac{1}{80}} \rightarrow a = \sqrt[4]{\ldots}$$

$$ax = \frac{\sqrt{x}}{-2x^2 + x + 1} \rightarrow -2ax^3 + ax^2 + ax - \sqrt{x} = 0 \xrightarrow{\text{مشتق}} -8ax^3 + 2ax + a - \frac{1}{2\sqrt{x}} = 0$$ ① ۹

$$f(\alpha) = \frac{\sqrt{\alpha}}{-2\alpha^2 + \alpha + 1} = a\alpha \rightarrow a\sqrt{\alpha}(-2\alpha^2 + \alpha + 1) = 1 \rightarrow -2a\alpha^{\frac{5}{2}} + a\alpha^{\frac{3}{2}} + a\alpha^{\frac{1}{2}} = 1$$

$$\xrightarrow{\text{مشتق}} -5a\alpha^{\frac{3}{2}} + \frac{3}{2}a\alpha^{\frac{1}{2}} + \frac{1}{2}a\alpha^{-\frac{1}{2}} = 0 \xrightarrow[\times 2\sqrt{\alpha}]{\div a} -10\alpha^2 + 3\alpha + 1 = 0 \rightarrow \begin{cases} \alpha = -\frac{1}{5} \\ \alpha = \frac{1}{2} \end{cases}$$

$$f(\alpha) = \frac{\sqrt{\frac{1}{2}}}{-2\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 1} = \frac{\sqrt{2}}{2}$$

خط $\rightarrow y = ax$ $\qquad A(\alpha, a\alpha)$

$$g'\left(\frac{\sqrt{5}}{2}\right) f'\left(g\left(\frac{\sqrt{5}}{2}\right)\right) \rightarrow g\left(\frac{\sqrt{5}}{2}\right) \quad f'(2) = ? \rightarrow 12 g'\left(\frac{\sqrt{5}}{2}\right) \rightarrow \boxed{-48\sqrt{5}}$$

$\hookrightarrow 3x^2 \rightarrow x = 2$ (۲)

۰/۵ ۱۰

$$g' = \frac{\frac{-2x}{2\sqrt{x^2-1}}}{x^2 - 1} = \frac{\frac{-\sqrt{5}}{2}}{\frac{\frac{1}{2}}{\frac{1}{4}}} = \boxed{-4\sqrt{5}}$$

$$\frac{-48\sqrt{5}}{-48\sqrt{5}} = 1$$

(جواب پایین صفحه)

سوال ۴

$$۳g'\left(\frac{۵\pi}{۳}\right)-f'\left(\frac{۵\pi}{۳}\right)=(۳g(x)-f(x))'\left(\frac{۵\pi}{۳}\right)$$

$$\rightarrow (۳g-f)(x)=\left(\frac{۹}{۳+\sin x}-\frac{۳-\sin^{۲}x}{۴-\sin^{۲}x}\right)=\frac{۹}{۳+\sin x}-\frac{(۳-\sin x)(۹+\sin^{۲}x+۳\sin x)}{(۳-\sin x)(۳+\sin x)}=-\sin x$$

$$\rightarrow (۳g-f)'(x)=-\cos x \rightarrow (۳g-f)'\left(\frac{۵\pi}{۳}\right)=-\cos\left(\frac{۵\pi}{۳}\right)=\frac{-۱}{۲}$$

سوال ۶

$$f(x)=x\,g(x)+۱ \rightarrow g(x)=\frac{f(x)-۱}{x} \rightarrow \lim_{x\to ۰} g(x)=\frac{۰}{۰} \xrightarrow{\text{hop}} \frac{+f'(۰)}{۰}=f'(۰)$$

$$f(x)=\left(\frac{-۱+\sin x}{۱+\sin x}\right)^{۲} \rightarrow f'(x)=۲\left(\frac{\cos x(۱+\sin x)-\cos x(-۱+\sin x)}{(۱+\sin x)^{۲}}\right)\times\left(\frac{-۱+\sin x}{۱+\sin x}\right)$$

$$\rightarrow f'(۰)=۲\times\left(\frac{۲}{۱}\right)\times(-۱)=-۴$$

سوال ۸

$$f(x)=۲\sqrt{x}(۲x^{۲}+۳)=۸x^{۲}\sqrt{x}+۶\sqrt{x} \rightarrow f'(x)=۲۰x\sqrt{x}+\frac{۳}{\sqrt{x}}=\frac{۲۰x^{۲}+۳}{\sqrt{x}}$$

$$y-۲\sqrt{\alpha}(۲\alpha^{۲}+۳)=\frac{۲۰\alpha^{۲}+۳}{\sqrt{\alpha}}(x-\alpha) \xrightarrow{(۰,۰)} -۲\sqrt{\alpha}(۲\alpha^{۲}+۳)=\frac{۲۰\alpha^{۲}+۳}{\sqrt{\alpha}}(-\alpha)$$

$$\rightarrow ۲(۲\alpha^{۲}+۳)=۲۰\alpha^{۲}+۳ \rightarrow ۱۲\alpha^{۲}=۳ \rightarrow \alpha^{۲}=\frac{۱}{۴}$$

$$m=\frac{۲۰\left(\frac{۱}{۴}\right)+۳}{\sqrt{\frac{۱}{۲}}}=۸\sqrt{۲}$$

سوال ۱۰

$$(f\circ g(\frac{\sqrt{۵}}{۲}))'=g'\left(\frac{\sqrt{۵}}{۲}\right)^{-}\times f'\left(g\left(\frac{\sqrt{۵}}{۲}\right)^{-}\right)$$

$$g(x)=(x^{۲}-۱)^{\frac{-۱}{۲}} \rightarrow g'(x)=\frac{-۱}{۲}(x^{۲}-۱)^{\frac{-۳}{۲}}\times ۲x \rightarrow g'\left(\frac{\sqrt{۵}}{۲}^{-}\right)=\frac{۱}{\sqrt{\left(\frac{۵}{۴}\right)^{-}-۱}}=\frac{۱}{\sqrt{\left(\frac{۱}{۴}\right)^{-}}}=\frac{۱}{\left(\frac{۱}{۲}\right)^{-}}=۲^{+}$$

$$f'(۲^{+})=\left((۲x)^{۳}\right)'=(۸x^{۳})'=۲۴x^{۲}=۲۴\times ۴$$

$$\rightarrow g'\left(\frac{\sqrt{۵}}{۲}^{-}\right)\times f'\left(g\left(\frac{\sqrt{۵}}{۲}\right)^{-}\right)=-۴\sqrt{۵}\times ۴\times ۲۴ \rightarrow \frac{۲۴\times ۴\times(-۴\sqrt{۵})}{-۴۸\sqrt{۵}}=۸$$