نام و نام خانوادگی دانیال حمدپرست........ پاسخنامه تشریحی تکلیف شماره ۲۴ کلاس دوازدهم ریاضی A

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$A(3,5)$ $\quad m = \frac{\Delta y}{\Delta x} = \frac{4}{3} \qquad d = \frac{4}{3}x + 1$

$B(0,1)$

$f'(3) = m = \boxed{\frac{4}{3}}$

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$(-1,1)$ $\quad m = \frac{\Delta y}{\Delta x} = \frac{1}{3} \qquad d = \frac{1}{3}x + \frac{4}{3}$

$(2,2)$

$f'(x) = \frac{a}{2\sqrt{ax-1}} = \frac{1}{3}$

$\frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1} \longrightarrow \frac{x^2}{9} + \frac{8}{9}x + \frac{16}{9} = ax - 1 \longrightarrow \frac{x^2}{9} + (\frac{8}{9} - a)x + \frac{25}{9} = 0$

$\xrightarrow{\times 9} x^2 + (8 - 9a)x + 25 = 0 \longrightarrow \Delta = 0 \longrightarrow (8-9a)^2 = 100 \longrightarrow \begin{cases} a = 2 \\ a = -\frac{2}{9} \text{ غق} \end{cases} \longrightarrow a = 2$

$f(x) = \sqrt{2x-1} \longrightarrow f(5) = \boxed{3}$

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$y = \frac{3}{4}x + \frac{n}{4}$

$f'(x) = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2}$

$f'(1) = \frac{8 + 4m - (2+m)}{16} = \frac{6+3m}{16} = \frac{3}{4} \longrightarrow 3m + 6 = 12 \longrightarrow m = 2$

$f(1) = \frac{2+m}{4} = 1 = \frac{3}{4} + \frac{n}{4} \longrightarrow n = 1$

$m + n = 2 + 1 = \boxed{3}$

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$g(x) = \frac{3}{3 + \sin x}$

$f(x) = \frac{\sin^2 x + 3\sin x + 9}{3 + \sin x}$

$(3g - f)(x) = \frac{-\sin^2 x - 3\sin x}{3 + \sin x} = -\sin x$

$(3g - f)'(x) = -\cos x$

$(3g - f)'(x) = -\cos\frac{5\pi}{3} = \boxed{-\frac{1}{2}}$

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$g(x) = \frac{1}{2x^5}$

$f(x) = -\frac{1}{\sqrt[5]{2x}}$

$f \circ g(x) = -\frac{1}{\sqrt[5]{2 \times \frac{1}{2x^5}}} = -x$

$(f \circ g(x))' = \boxed{-1}$

$f'(x) = g(x) + x g'(x) \rightarrow f'(0) = g(0)$

$f'(x) = 2 \times \left(\frac{-1+\sin x}{1+\sin x}\right) \times \left(\frac{\cos x(1+\sin x) - (-1+\sin x)(\cos x)}{(1+\sin x)^2}\right)$

$f'(0) = 2 \times (-1) \times 2 = -4 = g(0)$

$\lim_{x \to 0} g(x) = \boxed{-4}$

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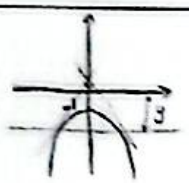

$y = -x^2 - 1 \rightarrow y' = -2x$

$x_1 = -x_2$

$(-2x_1) \times (-2x_2) = -1 \rightarrow x_1 x_2 = -\frac{1}{4}$ $\Big\} \rightarrow -x_2^2 = -\frac{1}{4} \rightarrow x_2 = \frac{1}{2}$, $x_1 = -\frac{1}{2}$

$y = -x^2 - 1 \xrightarrow{x = \pm\frac{1}{2}} y = -\frac{1}{4} - 1 = \boxed{-\frac{5}{4}}$

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$f(x) = 8x^{\frac{3}{2}} + 4x^{\frac{1}{2}} \rightarrow f'(x) = 20x^{\frac{3}{2}} + 2x^{-\frac{1}{2}} = m_d \rightarrow d: y = 20x^{\frac{3}{2}} + 2x^{\frac{1}{2}}$

$f(x) = d \rightarrow 8x^{\frac{3}{2}} + 4x^{\frac{1}{2}} = 20x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \rightarrow 2x^{\frac{1}{2}} = 12x^{\frac{3}{2}} \rightarrow 4x^{\frac{3}{2}} = x^{\frac{1}{2}}$

$\rightarrow 16x^5 - x = 0 \rightarrow \begin{cases} x = 0 \rightarrow y = 0 \\ x = \frac{1}{2} \rightarrow y = 2\sqrt{\frac{1}{2}} \times 4 = \frac{8}{\sqrt{2}} = 4\sqrt{2} \end{cases}$

$m = \frac{\Delta y}{\Delta x} = \frac{4\sqrt{2}}{\frac{1}{2}} = \boxed{8\sqrt{2}}$

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$f(x) = \frac{\sqrt{x}}{-2x^2 + x + 1} \rightarrow f'(x) = \frac{\frac{1}{2}x^{-\frac{1}{2}}(-2x^2+x+1) - x^{\frac{1}{2}}(-4x+1)}{(-2x^2+x+1)^2} = \frac{\frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}}{(-2x^2+x+1)^2} = m$

$d: y = \frac{\frac{3}{2}x^{\frac{3}{2}} - \frac{1}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}}}{(-2x^2+x+1)^2}$

$f(x) = d \rightarrow \frac{\frac{3}{2}x^{\frac{3}{2}} - \frac{1}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}}}{(-2x^2+x+1)^2} = \frac{x^{\frac{1}{2}}}{-2x^2+x+1} \rightarrow 5x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{3}{2}} - \frac{1}{2}x^{\frac{1}{2}} = 0 \quad x \neq \{-\frac{1}{2}, 1\}$

$\rightarrow \begin{cases} x^{\frac{1}{2}} = 0 \rightarrow x = 0 \\ 5x^2 - \frac{3}{2}x - \frac{1}{2} = 0 \rightarrow \begin{cases} x = \frac{1}{2} \rightarrow y = \frac{\sqrt{\frac{1}{2}}}{-\frac{1}{2} + \frac{1}{2} + 1} = \boxed{\frac{\sqrt{2}}{2}} \\ x = -\frac{1}{5} \text{ GGE} \end{cases} \end{cases}$

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$f(x) = x^3 \times [x]^3 \qquad f_{og}(x) = \left(\frac{1}{\sqrt{x^2-1}} \times \left[\frac{1}{\sqrt{x^2-1}}\right]\right)^3$

$g(x) = \frac{1}{\sqrt{x^2-1}}$

$(fog)'(x) = \lim_{h \to 0} \frac{f(x-h) - f(x)}{h} = \frac{0}{0} \xrightarrow{\text{hop}} -f'(x-h) - f'(x) = -2f'(x)$

$-2f'(x) = -2 \times 3 \times \left(\frac{1}{\sqrt{x^2-1}} \times \left[\frac{1}{\sqrt{x^2-1}}\right]\right)^2 \times \left(-\frac{1}{2}(x^2-1)^{-\frac{3}{2}} \times 2x \times \left[\frac{1}{\sqrt{x^2-1}}\right]\right)$

$-2f'\left(\frac{\sqrt{5}}{2}\right) = 12 \times 48\sqrt{5} \qquad \frac{12 \times 48\sqrt{5}}{-48\sqrt{5}} = \boxed{-12}$

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