"علیرضا آقاجانی"

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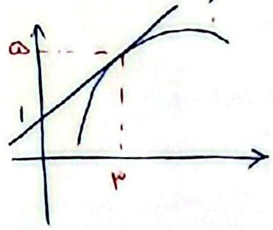

$\to$ نقطه تماس $\to \boxed{f(a)=g(a)}$ ، $\boxed{f'(a)=g'(a)}$

(1) f در نقطه تماس!

$\xrightarrow{x=3 \text{ در}} g'(a) \Rightarrow$ شیب خط $\Rightarrow \frac{y_2-y_1}{x_2-x_1}=m \to \frac{5-1}{3-0}=\frac{4}{3}$

$g(x)\to \frac{4}{3}x+1$

(2)

چون f در $x=3$ مماس g است مشتق هم در $x=3$ با شیب آن برابر است: $\frac{4}{3}$

(2)

خط مماس $\to \boxed{\frac{1}{3}x+\frac{4}{3}} = \boxed{g(x)}$

خط عبوری از ۲ نقطه

خط عبوری از ۲ نقطه $\to m=\frac{2-1}{2-(-1)}=\frac{1}{3} \xrightarrow{\text{پس}} f'(A)=\frac{1}{3}$

$\to f'(x)=\frac{a}{2\sqrt{ax-1}} \to f'(A)=\frac{a}{2\sqrt{Aa-1}}=\frac{1}{3} \to 3a=2\sqrt{Aa-1} \to \cancel{9a^2=4Aa-4}$

$\hookrightarrow 3a=2(\frac{1}{3}A+\frac{4}{3}) \to 3a=\frac{2}{3}A+\frac{8}{3}$

$\xrightarrow{f(A)=g(A)} \frac{1}{3}A+\frac{4}{3}=\sqrt{aA-1}$ $\quad \to \boxed{9a=2A+8}$

(2)

$\to \frac{a}{2\sqrt{\frac{9a^2-8a-2}{2}}}=\frac{1}{3} \to 3a=2\sqrt{\frac{9a^2-8a-2}{2}} \to 9a^2=18a^2-16a-4 \to \sqrt{\Delta}=20$

$a=\frac{16\pm 20}{18}$ $\to \boxed{2}$ قق ، $\to -\frac{2}{9}$ غق

$f(5)=\sqrt{2x-1} \to \sqrt{(2\times 5)-1}=3$

(3)

$y' \to \frac{(2x+m)(x+3)-x^2-mx-1}{(x+3)^2} \to \frac{2x^2+\cancel{mx}+6x+3m-x^2-\cancel{mx}-1}{(x+3)^2}=\frac{x^2+6x+3m-1}{(x+3)^2}$

(2)

$\to y'(1)=\frac{\cancel{6+3m}}{\cancel{16}}=\frac{\cancel{3}}{\cancel{4}} \to 2+m=4 \to \boxed{m=2}$

$4y-3x=n \to m=-\frac{a}{b} \to \frac{3}{4}$

$m+n=1+2=3$

$y(1)=\frac{1+2+1}{4}=1$ و $(4\times 1)-(3\times 1)=n \to 1$

(4)

$(3g-f)'(\frac{5\pi}{3}) \xrightarrow{3g-f=?} \frac{9}{3+\sin x}-\frac{27-\sin^3 x}{\underbrace{9-\sin^2 x}} \to \frac{\overbrace{-\sin^2 x-3\sin x}^{-\sin x(\sin x+3)}}{3+\sin x}=-\sin x$

$\frac{\cancel{(3-\sin x)}(9+\sin^2 x+3\sin x)}{\cancel{(3-\sin x)}(3+\sin x)}$

(2)

$(-\sin x)' \to \boxed{-\cos x} \xrightarrow{x=\frac{5\pi}{3}} -\cos\frac{5\pi}{3}=-\frac{1}{2}$

(5)

$g'(\sqrt[5]{3})\,f'(g(\sqrt[5]{3})) \to (f\circ g)'(\sqrt[5]{3}) \xrightarrow[x=\sqrt[5]{3},\ x>0]{f\circ g=?} -\frac{1}{5\sqrt{\frac{1}{x^5+10x^5}}+\left|\frac{1}{x^5+10x^5}\right|}=-\frac{1}{\sqrt{\frac{1}{x^5}}}=-x$

(2)

$(-x)' \to -1$

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$x\to 0$ چون $\to$ $\boxed{\sin x \xrightarrow{\text{هم‌ارزی}} x}$

$$f(x)=\left(\frac{x-1}{x+1}\right)^2 \to \left(\frac{x-1}{x+1}\right)^2 = x\big(g(x)\big)+1 \to \frac{x^2-2x+1}{x^2+2x+1} = xg(x)+1$$

$$\to \frac{\cancel{x^2}-2x+1\cancel{-x^2}-2x-1}{(x^2+2x+1)(x)} = g(x) \to \boxed{\frac{-4\cancel{x}}{\cancel{x}(x^2+2x+1)} = g(x)}$$

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$$\lim_{x\to 0} g(x) = \frac{-4}{1} = -4$$

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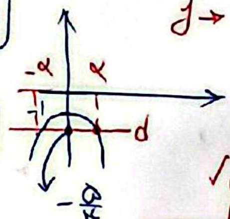
$$y=x^2+1 \xrightarrow[(x,-1)]{\text{قرینه نسبت به } x} (-x^2-1)$$

$\boxed{x\le 0}$

$\boxed{y\le -1}$



$y' \to -2x$ → -2α , 2α ; $\cancel{x} + 4\alpha^2 = +1$ → $\boxed{\alpha = \pm\frac12}$

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همون عرض از مبدأ خط d ($y=k$)

$$\longrightarrow y=-x^2-1 \xrightarrow{x=\frac12} -\frac14-1 = -\frac54$$

$\rightsquigarrow$ $\boxed{\text{فاصله خط d از مبدأ} \to -\frac54}$

(فاصله به صورت مثبت بیان می‌شود)

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$\cancel{d(\alpha)=}$ $d(\alpha)=f(\alpha) \to a\alpha = 2\sqrt\alpha\,(4\alpha^2+3)$ (II) $\qquad y=ax \leftarrow d$

$$d'(\alpha)=f'(\alpha) \to a = \frac{4x^2+3}{\sqrt x} + 16x\sqrt x \to \frac{4x^2+3+16x^2}{\sqrt x}$$

$\boxed{f'(a)=\left(\frac{20\alpha^2+3}{\sqrt\alpha}\right)}$ III

(I) دو طرف در α ضرب شود

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$$\xrightarrow{I,II} 2\sqrt\alpha\,(4\alpha^2+3) = \frac{20\alpha^3+3\alpha}{\sqrt\alpha} \to 2\cancel{\alpha}(4\alpha^2+3) = (20\alpha^2+3)\cancel{\alpha} \to 8\alpha^2+6 = 20\alpha^2+3$$

$$\to 12\alpha^2=3 \to 4\alpha^2=1 \to \boxed{\alpha = \pm\frac12}$$

قق ، غ‌قق

$$\xrightarrow{III} a = \frac{(20\times\frac14)+3}{\sqrt{\frac12}} = 8\sqrt2$$

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$$d(x)=f(x) \xrightarrow{x=A} aA = \frac{\sqrt A}{-2A^2+A+1} \xrightarrow{\div A} \text{(I)}$$

(جواب پایین صفحه)

$\boxed{ax \leftarrow d}$

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$$d'(x)=f'(x) \xrightarrow{x=A} a = \frac{\frac{-2A^2+A+1}{2\sqrt A} + (\cancel{-4A+1})(\sqrt A)}{(2A+1)^2(-A+1)^2}$$

$+4A\sqrt A - \sqrt A$ ، $9A^2-A+1$

$$\to a = \frac{-2A^2+A+1+8A^2-2A}{(2A+1)^2(1-A)^2} \quad \text{(II)}$$

$$\xrightarrow{I,II} \frac{9A^2-A+1}{\underbrace{(2A+1)^2(1-A)^2}_{-2A^2+A+1}} = \frac{\sqrt A}{\underbrace{-2A^3+A^2+A}_{\cancel{A(2A+1)(1-A)}}} \to 9A^3-A^2+A = -2\sqrt A A^2 + A\sqrt A + \sqrt A$$

$$\to 9A^2\sqrt A - A\sqrt A + \sqrt A = -2A^2+A+1$$

$\boxed{g'(x) = \frac{\frac{-1}{\sqrt{x^2-1}}}{x^2-1} \to g'\!\left(\frac{\sqrt5}{2}^-\right) = -8}$

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$$\lim_{x\to\left(\frac{\sqrt5}{2}\right)^-} \frac{-fog\left(\frac{\sqrt5}{2}\right)+fog(x)}{x-\frac{\sqrt5}{2}} \rightsquigarrow \frac{fog(x)-96}{x-\frac{\sqrt5}{2}} \rightsquigarrow \frac{fog\left(\frac{\sqrt5}{2}^-\right)-96}{x-\frac{\sqrt5}{2}} \rightsquigarrow \frac{f(2^+)-96}{a-\frac{\sqrt5}{2}} = \frac00$$

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$$\xrightarrow{\text{Hop}} \frac{g'\left(\frac{\sqrt5}{2}^-\right)\times f'\left(g\,\frac{\sqrt5}{2}\right)}{1} \to \frac{-8\times f'(2^+)}{1} \to \frac{(-8)\times(24\times4)}{1} = -32\times24 \xrightarrow[\div]{} \frac{24\times\cancel{-32}^{\,-4}}{\cancel{4}\,8\sqrt5} = \frac{-96\sqrt5}{5}$$

(جواب پایین صفحه)

Scanned with CamScanner

سوال ۹

d خط $\to y = ax \qquad A(\alpha, a\alpha)$

$$f(\alpha) = \frac{\sqrt{\alpha}}{-2\alpha^2+\alpha+1} = a\alpha \to a\sqrt{\alpha}(-2\alpha^2+\alpha+1) = 1 \to -2a\alpha^{\frac{5}{2}} + a\alpha^{\frac{1}{2}} + a\alpha^{\frac{3}{2}} = 1$$

$$\xrightarrow{\text{مشتق}} -5a\alpha^{\frac{3}{2}} + \frac{3}{2}a\alpha^{\frac{1}{2}} + \frac{1}{2}a\alpha^{-\frac{1}{2}} = 0 \xrightarrow[\times 2\sqrt{\alpha}]{\div a} -10\alpha^2 + 3\alpha + 1 = 0 \longrightarrow \begin{cases} \alpha = \frac{-1}{5} \\ \alpha = \frac{1}{2} \end{cases}$$

$$f(\alpha) = \frac{\sqrt{\frac{1}{2}}}{-2(\frac{1}{2})^2 + \frac{1}{2} + 1} = \frac{\sqrt{2}}{2}$$

سوال ۱۰

$$\left(fog\left(\frac{\sqrt{5}}{2}\right)\right)' = g'\left(\frac{\sqrt{5}}{2}\right)^- \times f'\left(g\left(\frac{\sqrt{5}}{2}\right)^-\right)$$

$$g(x) = (x^2-1)^{-\frac{1}{2}} \to g'(x) = \frac{-1}{2}(x^2-1)^{-\frac{3}{2}} \times 2x \longrightarrow g'\left(\frac{\sqrt{5}}{2}^-\right) = \frac{1}{\sqrt{(\frac{5}{4})^- - 1}} = \frac{1}{\sqrt{(\frac{1}{4})^-}} = \frac{1}{(\frac{1}{2})^-} = 2^+$$

$$f'(2^+) = ((2x)^3)' = (8x^3)' = 24x^2 = 24 \times 4$$

$$\longrightarrow g'\left(\frac{\sqrt{5}}{2}^-\right) \times f'\left(g\left(\frac{\sqrt{5}}{2}\right)^-\right) = -4\sqrt{5} \times 4 \times 24 \longrightarrow \frac{24 \times 4 \times (-4\sqrt{5})}{-48\sqrt{5}} = 8$$