14

امیر مهدی اسماعیلی فرد دوازدهم سری A

۱-

معادله خط: $y = \frac{4}{3}x + 1$

گذرا از نقطه: $A\begin{vmatrix}3\\5\end{vmatrix}, B\begin{vmatrix}0\\1\end{vmatrix}$

$f'(3) =$ شیب خط مماس بر تابع f در نقطه $x=3$ $= \boxed{\frac{4}{3}}$

$m = \frac{\Delta y}{\Delta x} = \frac{5-1}{3-0} = \frac{4}{3}$

2

۲-

$\begin{cases}(-1,1)\\(2,2)\end{cases}$ $m = \frac{2-1}{2+1} = \frac{1}{3}$, $y = \frac{1}{3}x + \frac{4}{3}$

$\xrightarrow{\text{معادله تلاقی}} \frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1} \rightarrow x^2 + 8x + 16 = 9ax - 9$

$\rightarrow x^2 + (8-9a)x + 25 = 0$

$f(x) = \sqrt{2x-1} \xrightarrow{x=5} f(5) = \sqrt{9} = \boxed{3}$

$\xrightarrow{\Delta = 0} \begin{cases}a = 2 \checkmark\\ a = -\frac{2}{9} \quad \text{غ ق ق}\end{cases}$

2

۳-

خط مماس: $4y - 3x = n \rightarrow y = \frac{3x+n}{4}$

معادله تلاقی: $\frac{3x+n}{4} = \frac{x^2+mx+1}{x+3} \rightarrow 0 = x^2 + (\underbrace{4m-n-9}_{-2})x + \underbrace{4-3n}_{1}$

معادله ریشه مضاعف $x=1$ دارد $\Rightarrow 0 = (x-1)^2 = x^2 - 2x + 1$

تساوی $\begin{cases}4-3n=1\\4m-n-9=-2\end{cases} \Rightarrow \begin{matrix}m=2\\n=1\end{matrix} \Rightarrow \boxed{m+n=3}$

2

۴-

$(3g-f)'(x) = ?$

$\frac{9}{3+\sin x} - \frac{9+\sin^2 x+3\sin x}{\sin x+3} = \frac{-\sin x(\sin x+3)}{\sin x+3} = (3g-f)(x)$

$= -\sin x$

$\longrightarrow (3g-f)'(x) = (-\sin x)' = -\cos x \longrightarrow (3g-f)'(\frac{5\pi}{3}) = -\cos\frac{5\pi}{3} = \boxed{-\frac{1}{2}}$

2

۵-

$fog(x) = \frac{-1}{\sqrt[5]{g(x)+|g(x)|}} = \frac{-1}{\sqrt[5]{2g(x)}} = \frac{-1}{\sqrt[5]{\cdots\frac{1}{x^5}}} = -x$

$f(x) = \begin{cases}\alpha & x \le 0\\ \frac{-1}{\sqrt[5]{2x}} & x > 0\end{cases}$

$|fog(x)| = -1 \longrightarrow$ $\boxed{-1 = \text{جواب سؤال}}$

$g(x) = \begin{cases}9 & x \le 0\\ \frac{1}{2x^5} & x > 0\end{cases}$

2

1

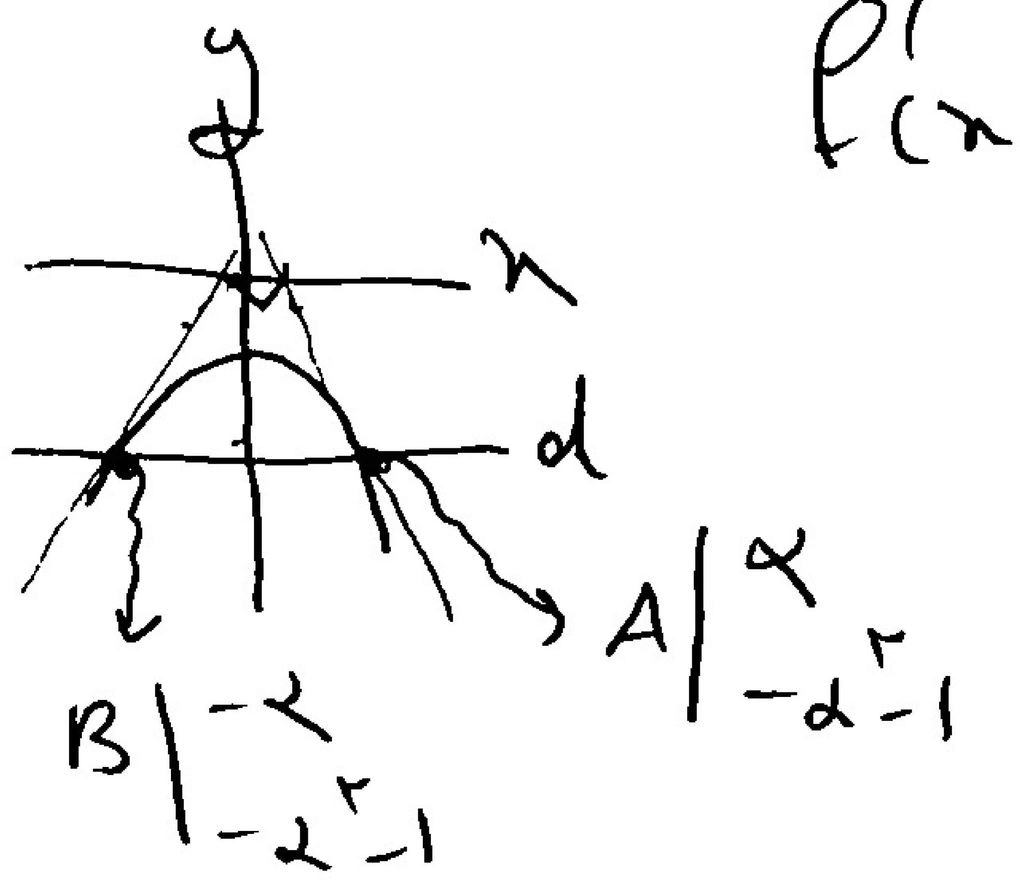
$$g(x)=\frac{f(x)-1}{x}=\frac{\left(\frac{\sin x-1}{\sin x+1}\right)^2-1}{x}$$ — 6

(در همسایگی ۰ هم‌ارزی می‌ریزیم)

$$\lim_{x\to 0} g(x)=\lim_{x\to 0}\frac{\left(\frac{x-1}{x+1}\right)^2-1}{x}=\lim_{x\to 0}\frac{-4}{(x+1)^2}=\frac{-4}{1}=\boxed{-4}$$

(2)

تابع قرینه نسبت به محور x = $f(x)=-x^2-1$



$$f'(x)=-2x \to f'(\alpha)=-2\alpha,\quad f'(-\alpha)=+2\alpha$$ — 7

عمود $mm'=-1$

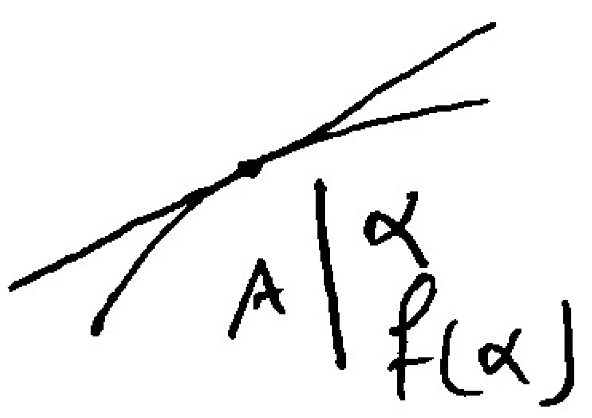
$$-2\alpha\times 2\alpha=-1 \xrightarrow{\alpha>0} \alpha=\frac{1}{2}$$

(2)

$$f(\alpha)=\frac{-5}{4}$$

فاصله خط d تا مبدأ برابر $\frac{5}{4}$ می‌باشد

$$d:\ y=mx \longrightarrow m=f'(\alpha)=\frac{f(\alpha)-0}{\alpha-0}$$ — 8



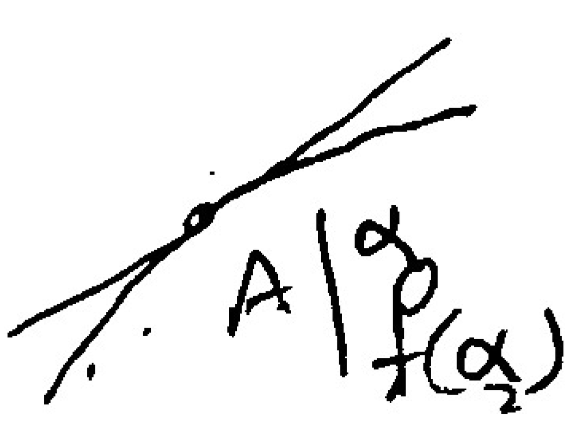
$$\to m=\frac{4\alpha^2+3}{\sqrt{\alpha}}+16\alpha\sqrt{\alpha}=\frac{2\sqrt{\alpha}\,(4\alpha^2+3)}{\alpha}$$

$$\longrightarrow \frac{4\alpha^2+3}{\sqrt{\alpha}}+\frac{16\alpha^2}{\sqrt{\alpha}}=\frac{8\alpha^2+6}{\sqrt{\alpha}}$$

(2)

$$\longrightarrow \alpha^2=\frac{1}{4}\to \alpha=\frac{-1}{2}$$ غ.ق.ق

$$\alpha=+\frac{1}{2}\ ✓ \to m=f'\!\left(\tfrac{1}{2}\right)=\boxed{8\sqrt{2}}$$



$$m=f'(\alpha)=\frac{f(\alpha)}{\alpha}$$

$$\longrightarrow \alpha f'(\alpha)=f(\alpha)\to \alpha\times\frac{6\alpha^2-\alpha+1}{2\sqrt{\alpha}\,(-2\alpha^2+\alpha+1)^2}=\frac{\sqrt{\alpha}}{-2\alpha^2+\alpha+1}$$ (2) — 9

$$\longrightarrow \frac{6\alpha^2-\alpha+1}{-4\alpha^2+2\alpha+2}=1\longrightarrow 10\alpha^2-3\alpha-1=0$$

$\alpha=0.5$ ✓ $\longrightarrow f(\alpha)=f(0.5)=\boxed{\frac{\sqrt{2}}{2}}$

$\alpha=-0.2$ در دامنه نیست

مشتق‌پذیر تابع $f(x)$ در همسایگی $\frac{\sqrt{5}}{2}$ [illegible]: $f(x)=x^3$

$$(f\circ g)'(x)=f'\!\left(\frac{1}{\sqrt{x^2-1}}\right)\times -\frac{1}{2}(x^2-1)^{-\frac{3}{2}}(2x)$$ — 10

$$\xrightarrow{x=\frac{\sqrt{5}}{2}} (f\circ g)'\!\left(\tfrac{\sqrt{5}}{2}\right)=f'(2)\times-\frac{1}{2}\times 8\times\sqrt{5}=\frac{15}{4}\times-\frac{1}{2}\times 8\sqrt{5}=\boxed{-15\sqrt{5}}$$

(1)

سوال ۱۰

$(f\circ g(\frac{\sqrt{۵}}{۲}))' = g'(\frac{\sqrt{۵}}{۲})^- \times f'(g(\frac{\sqrt{۵}}{۲})^-)$

$g(x) = (x^۲-۱)^{-\frac{۱}{۲}} \to g'(x) = \frac{-۱}{۲}(x^۲-۱)^{-\frac{۳}{۲}} \times ۲x \to g'(\frac{\sqrt{۵}}{۲}^-) = \frac{۱}{\sqrt{(\frac{۵}{۴})^- - ۱}} = \frac{۱}{\sqrt{(\frac{۱}{۴})^-}} = \frac{۱}{(\frac{۱}{۲})^-} = ۲^+$

$f'(۲^+) = ((۲x)^۳)' = (۸x^۳)' = ۲۴x^۲ = ۲۴ \times ۴$

$\to g'(\frac{\sqrt{۵}}{۲}^-) \times f'(g(\frac{\sqrt{۵}}{۲})^-) = -۴\sqrt{۵} \times ۴ \times ۲۴ \to \frac{۲۴ \times ۴ \times (-۴\sqrt{۵})}{-۴۸\sqrt{۵}} = ۸$