

$$f'(x) = \frac{\Delta y}{\Delta x} = \frac{x-1}{x-0} = \frac{x}{x}$$

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$$\text{نسبت خط مماس} = \frac{\Delta y}{\Delta x} = \frac{x-1}{x+1} = \frac{1}{x} \rightarrow f(x) = \frac{a}{\sqrt{x+1}} = \frac{1}{x}$$

$$\rightarrow \sqrt{x+1} = x \rightarrow$$

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$$\text{نسبت خط مماس} = \frac{x}{x} \rightarrow f'(1) = \frac{x}{x} \rightarrow f'(x) = \frac{(x+m)(x+x) - (x^2+m+1)}{(x+x)^2}$$

$$\rightarrow f'(1) = \frac{x(x+m)}{4} = \frac{x}{x} \rightarrow \boxed{m=2} \quad f(x) = \frac{(x+1)^2}{(x+x)} \rightarrow f(1) = \frac{x}{x} = 1$$

$$\rightarrow y = \frac{x}{x}x + n \xrightarrow{x=1} 1 = \frac{x}{x} + n \rightarrow \boxed{n = \frac{1}{x}} \Rightarrow m+n = \frac{3}{x}$$

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$$f(x) = \frac{(x-\sin x)(9+x\sin x + \sin^2 x)}{(x-\sin x)(x+\sin x)} \rightarrow f'(x) = \frac{(\sin^2 x + x\sin x)(\cos x)}{(\sin x + x)^2}$$

$$g'(x) = x \left(-\frac{\cos x}{(x+\sin x)^2} \right) \rightarrow g'\left(\frac{\pi}{2}\right) = \frac{-x}{x^2 - 4\sqrt{x}} \quad , \quad f'\left(\frac{\pi}{2}\right) = \frac{x\sqrt{x}-1}{-x(x-4\sqrt{x})}$$

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$$\Rightarrow \text{جواب} = -\frac{1}{x}$$

$$g(x) = \frac{1}{x} x^{-a} \rightarrow g'(x) = -\frac{a}{x} x^{-a} \rightarrow g'(a\sqrt{x}) = -\frac{a}{x} \times \frac{1}{a\sqrt{x^a}} \quad , \quad g(a\sqrt{x}) = \frac{1}{x}$$

$$f'(g(a\sqrt{x})) = f'\left(\frac{1}{x}\right) \quad , \quad f(x) = \frac{1}{g\sqrt{x}} \rightarrow f'(x) = \frac{x}{a^a \sqrt{(x+1)^a}} \rightarrow f'\left(\frac{1}{x}\right) = \frac{x}{a^a \sqrt{x^a}}$$

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$$\Rightarrow \text{جواب} = -1$$

$$f'(x) = \lim_{h \rightarrow 0} \left(\frac{\sin(x+h) - \sin(x)}{h} \right) \left(\frac{(\cos(x+h))(\sin(x+h)) - (\cos(x))(\sin(x))}{(\sin(x+h))^2} \right) = \lim_{h \rightarrow 0} \frac{(\sin(x)-1)\cos(x)}{(\sin(x)+1)^2}$$

$$\rightarrow f'(x) = g(x) + xg'(x) \rightarrow f'(0) = g'(0) = \lim_{x \rightarrow 0} \frac{-1}{1} = -1 = \lim_{x \rightarrow 0} g(x)$$

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$$f(x) = -x^2 - 1 \rightarrow f'(x) = -2x = 1 \rightarrow x = -\frac{1}{2} \rightarrow \text{خط مماس در } x = -\frac{1}{2} \text{ و } \frac{1}{2} \text{ در } d$$

$$\rightarrow f\left(\frac{1}{2}\right) = -\frac{1}{4} - 1 = -\frac{5}{4} \rightarrow \text{جواب} = \frac{5}{4}$$

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$$f(x) = \sqrt{\frac{1}{x}} (x^2 + 1) + \sqrt{x} (1/x) \rightarrow \frac{f(x)}{x} = f'(x)$$

$$\dots \text{حساب} \Rightarrow x = \frac{1}{2} \rightarrow f\left(\frac{1}{2}\right) = 1\sqrt{2} = \text{خط مماس}$$

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$$f'(x) = \frac{\left(\frac{1}{\sqrt{x}}\right)(-2x^2 + x + 1) - \sqrt{x}(-2x + 1)}{(-2x^2 + x + 1)^2} \rightarrow \frac{f(x)}{x} = f'(x)$$

$$\dots \text{حساب} \rightarrow x = \frac{1}{2} \rightarrow f\left(\frac{1}{2}\right) = \frac{\sqrt{\frac{1}{2}}}{-\frac{1}{2} + \frac{1}{2} + 1} = \frac{\sqrt{\frac{1}{2}}}{1}$$

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تابع $f(x)$ برای $x = \frac{\sqrt{5}}{2}$ می تواند برابر با x^2 باشد پس:

$$(f \circ g)'(x) = f'(g(x)) \times g'(x) = \frac{2}{x^2 - 1} \times \left(\frac{2x}{2\sqrt{x^2 - 1}} \right) \rightarrow (f \circ g)'\left(\frac{\sqrt{5}}{2}\right) =$$

$$= \frac{2}{\frac{5}{4} - 1} \times \left(-\frac{\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}} \right) = -2\sqrt{5} \rightarrow \text{یک برابر}$$

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