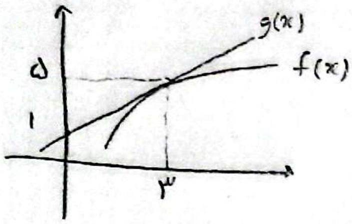
نام و نام خانوادگی: حسام رفیع زاده　　«به نام خدا»　　مقطع: دوازدهم پسر A

شماره: ۲۴

-۱

$f'(3)$ برابر است با شیب خط $g(x)$ داریم:

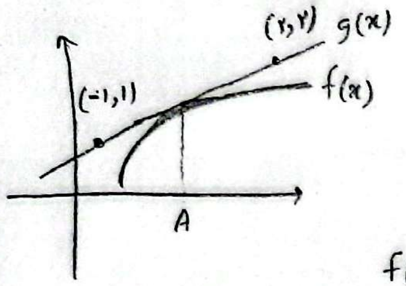


$$m_{g(x)} = \frac{\Delta y}{\Delta x} \to m = \frac{4}{3}$$

$$\to f'(3) = \frac{4}{3}$$

-۲

$f'(A)$ برابر شیب خط $g(x)$ است داریم:



$$m_{g(x)} = \frac{\Delta y}{\Delta x} \to m = \frac{1}{3} \quad ① \to g(x) = \frac{1}{3}x + \frac{4}{3}$$

$$f'(A) = \frac{1}{3} \quad ①$$

$$f(x) = g(x) \xrightarrow[\Delta = 0]{\text{چون مماس اند}} \sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3}$$

$$\xrightarrow{()^2} ax - 1 = \frac{1}{9}x^2 + \frac{16}{9} + \frac{8x}{9} \to \frac{1}{9}x^2 + \left(\frac{8}{9} - a\right)x + \frac{25}{9} = 0$$

$$\xrightarrow{\Delta = 0} \left(\frac{8}{9} - a\right)^2 = \frac{100}{81} \begin{cases} \frac{8}{9} - a = \frac{10}{9} \to a = -\frac{2}{9} \notin D_f \\ \frac{8}{9} - a = \frac{-10}{9} \to \boxed{a = 2 \checkmark} \end{cases}$$

$$f(x) = \sqrt{2x-1} \xrightarrow{x=5} f(5) = \sqrt{9} = \boxed{3}$$

-۳

$$y = \frac{x^2 + mx + 1}{x+3} \xrightarrow{()'} y' = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} \to y' = \frac{x^2 + 6x + 3m - 1}{(x+3)^2}$$

$$\xrightarrow{x=1} f'(1) = \frac{3m+6}{16} \xrightarrow[\text{برابر است}]{\text{با شیب خط داده شده}} \frac{3m+6}{16} = \frac{3}{4} \to m + 2 = 4 \to \boxed{m = 2}$$

$$f(1) = \text{معادلهٔ خط مماس به ازای } x=1 \to 1 = \frac{3}{4} + \frac{n}{4} \to \frac{n}{4} = \frac{1}{4} \to \boxed{n = 1}$$

$$m + n = \boxed{2}$$

۴-

$$f(x)=\frac{۲۷-\sin^{۳}x}{۹-\sin^{۲}x}\ ,\ g(x)=\frac{۳}{۳+\sin x}\ ,\ \left(۳g\left(\frac{۵\pi}{۳}\right)-f\left(\frac{۵\pi}{۳}\right)\right)'=?$$

$$۳g(x)-f(x)=\frac{۹}{۳+\sin x}-\frac{۲۷-\sin^{۳}x}{۹-\sin^{۲}x}=\frac{۹(۳-\sin x)-۲۷+\sin^{۳}x}{۹-\sin^{۲}x}$$

$$=\frac{\sin^{۳}x-۹\sin x}{۹-\sin^{۲}x}\xrightarrow[\text{بعنوان fog ببین}]{(\ )'}\frac{(۳x^{۲}-۹)(۹-x^{۲})+۲x(x^{۳}-۹x)}{(۹-x^{۲})^{۲}}\times\cos x$$

$$\frac{-x^{۴}+۱۸x^{۲}-۸۱}{-(-x^{۴}+۱۸x^{۲}-۸۱)}\times\cos x=-\cos x\xrightarrow{x=\frac{۵\pi}{۳}}-\cos\left(\frac{\pi}{۳}\right)=\boxed{\frac{-۱}{۲}}$$

۵-

$$f(x)=-\frac{۱}{\sqrt[۵]{x+|x|}}\ ,\ g(x)=\frac{۱}{x^{۵}+|x^{۵}|}\ ,\ (fog(x))'=?$$

$$fog(x)=\frac{-۱}{\sqrt[۵]{\frac{۱}{x^{۵}+|x^{۵}|}+\left|\frac{۱}{x^{۵}+|x^{۵}|}\right|}}\overset{\substack{x>۰\\ \text{در دامنه}}}{=}-x\rightarrow fog(x)=-x\xrightarrow{(\ )'}\boxed{(fog(x))'=-۱}$$

۶-

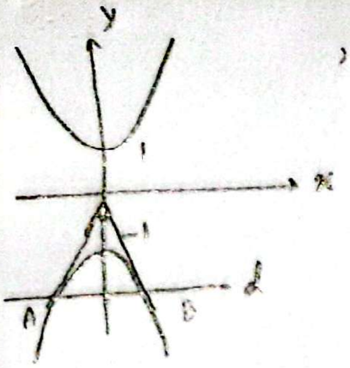
$$f(x)=\left(\frac{-۱+\sin x}{۱+\sin x}\right)^{۲}\ ,\ f(x)=x\,g(x)+۱\rightarrow g(x)=\frac{f(x)-۱}{x}\ ,\ \lim_{x\to ۰}g(x)=?$$

$$\lim_{x\to ۰}\frac{f(x)-۱}{x}=\frac{۰}{۰}\xrightarrow{H}\lim_{x\to ۰}\frac{f'(x)}{۱}=f'(x)=f'(۰)$$

$$f(x)=g\circ h(x)\rightarrow\begin{cases}h(x)=\sin x\\ g(x)=\left(\frac{x-۱}{x+۱}\right)^{۲}\end{cases}\xrightarrow{(\ )'}۲\times\frac{\sin x-۱}{\sin x+۱}\times\frac{۲}{(\sin x+۱)^{۲}}\times\cos x\xrightarrow{x=۰}$$

$$f'(۰)=۲\times\frac{-۱}{۱}\times\frac{۲}{۱}\times ۱=\boxed{-۴}$$

۷-



تابع ایجاد شده: $y = -x^{۲} - ۱ \xrightarrow{()'} y' = -۲x \begin{cases} x=A & y' = -۲A \\ x=B & y' = -۲B \end{cases} \xrightarrow[\text{عمود}]{\text{برهم}}$

$-۲A \times -۲B = -۱ \to A \times B = \frac{-۱}{۴}$ ①

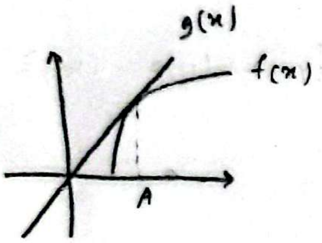
$y = -x^{۲} - ۱ \xrightarrow{x=A,\,B} \cancel{-}A^{۲} \cancel{-۱} = \cancel{-}B^{۲} \cancel{-۱} \to A = \pm B \xrightarrow[\text{به شکل}]{\text{با توجه}} A = -B$ ②

$①,② \to B^{۲} = \frac{۱}{۴} \to B = \frac{۱}{۲} , A = \frac{-۱}{۲}$

$\to y = -x^{۲} - ۱ \xrightarrow{x=\frac{۱}{۲}} y = -\frac{۱}{۴} - ۱ = \frac{-۵}{۴} \to$ فاصلهٔ d از مبدأ: $\boxed{\frac{-۵}{۴}}$

۸-

$f(x) = ۲\sqrt{x}\,(۴x^{۲}+۳)$, خط d : $g(x) = ax$



$f(x) = ۸x^{\frac{۵}{۲}} + ۶x^{۱/۲} \xrightarrow{()'} f'(x) = ۲۰\,x^{۳/۲} + ۳x^{\frac{-۱}{۲}}$

$g(x) = ax$

$f(A) = g(A) \to ۸A^{\frac{۵}{۲}} + ۶A^{۱/۲} = aA \xrightarrow{\div A} ۸A^{۳/۲} + ۶A^{-۱/۲} = a$ ①

$f'(A) = g'(A) \to ۲۰A^{۳/۲} + ۳A^{-۱/۲} = a$

$\to ۲۰A^{\frac{۳}{۲}} + ۳A^{-۱/۲} = ۸A^{۳/۲} + ۶A^{-۱/۲} \to ۱۲A^{۳/۲} = ۳A^{-۱/۲} \to ۴A^{۳/۲} = A^{-۱/۲}$

$\to ۴ = A^{-۲} \to ۴ = \frac{۱}{A^{۲}} \to A = ۱/۲ \to A = ۲^{-۱}$

$① \to ۸ \times ۲^{\frac{-۳}{۲}} + ۶ \times ۲^{۱/۲} = a \to \boxed{a = ۸\sqrt{۲}}$

Scanned with CamScanner

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$$f(x)=\frac{\sqrt{x}}{-2x^2+x+1} \quad , \quad \text{خط } d : \; y = ax = g(x)$$

$$f(A)=g(A) \rightarrow \frac{A^{1/2}}{-2A^2+A+1} = aA \rightarrow \frac{A^{-1/2}}{-2A^2+A+1} = a \quad ①$$

$$f'(x) = \frac{\frac{1}{2}x^{-1/2}(-2x^2+x+1) - (-4x+1)x^{1/2}}{(-2x^2+x+1)^2} = \frac{3x^{3/2}+\frac{1}{2}x^{-1/2}-\frac{1}{2}x^{1/2}}{(-2x^2+x+1)^2}$$

$$f'(A) = \frac{3A^{3/2}+\frac{1}{2}A^{-1/2}-\frac{1}{2}A^{1/2}}{(-2A^2+A+1)^2} = a \quad ②$$

$$\rightarrow 3A^{3/2}+\frac{1}{2}A^{-1/2}-\frac{1}{2}A^{1/2} = A^{-1/2}(-2A^2+A+1)$$

$$\rightarrow 3A^2+\frac{1}{2}-\frac{1}{2}A = -2A^2+A+1 \rightarrow 5A^2-\frac{3}{2}A-\frac{1}{2}=0$$

$$\xrightarrow{\times 2} 10A^2-3A-1=0 \xrightarrow[\text{روش}]{\text{روش}} A^2-3A-10=0 \rightarrow (\underset{5}{A-5})(\underset{-2}{A+2})=0$$

$$\rightarrow A=-\frac{1}{5}\ ,\ \frac{1}{2} \xrightarrow{D_f} A=\frac{1}{2} \rightarrow A=2^{-1}$$

$$① \rightarrow \frac{2^{1/2}}{-2\times 2^{-2}+2^{-1}+1} = a \rightarrow \frac{\sqrt{2}}{\cancel{\frac{1}{2}}-\cancel{\frac{1}{2}}+1} \rightarrow \underset{\text{شیب}}{\boxed{a=\sqrt{2}}}$$

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$$(f\circ g)'_-\left(\frac{\sqrt{5}}{2}\right)=? \ ,\ f\circ g(x) = \left(g(x)\cdot[g(x)]\right)^3 \xrightarrow{()'} 3\left(g(x)\cdot[g(x)]\right)^2\times\left(g'(x)\cdot[g(x)]+0\right)$$

$$g\left(\frac{\sqrt{5}}{2}^-\right)=2^+ \ ①\ ,\ g(x)=\frac{1}{\sqrt{x^2-1}} \xrightarrow{()'} \frac{-\cancel{2}x}{\cancel{2}\sqrt{x^2-1}\times(x^2-1)} \xrightarrow{x=\frac{\sqrt{5}}{2}} \frac{-\frac{\sqrt{5}}{\cancel{2}}}{\frac{1}{\cancel{2}}\times\frac{1}{4}} = -4\sqrt{5} \ ②$$

$$\left(f\circ g\left(\frac{\sqrt{5}}{2}\right)\right)'_- \underset{①,②}{=} 3(\underbrace{2\times 2}_{4})^2\times(-4\sqrt{5}\times 2) = -16\times 3\times 8\sqrt{5} \xrightarrow{\div\,-48\sqrt{5}} \frac{-16\times 3\times 8\cancel{\sqrt{5}}}{-\underset{2^4\times 3}{48}\cancel{\sqrt{5}}} = \boxed{8}$$