

گشت متوسط  $\frac{f(3)-f(1)}{3-1} = \frac{1-\frac{a}{3}-(1-a)}{2} = \frac{\frac{2a}{3}}{2} = \frac{a}{3}$

گشت  $f'(x) = \frac{a}{2^x} = \frac{a}{3} \rightarrow x=3$  ✓ (2)

$y = 2ax^2 - \omega x + 11a \rightarrow y_A = 2a x_A^2 - \omega x_A + 11a = x_A \xrightarrow{I} 2a(\frac{x}{2a})^2 - \omega(\frac{x}{2a}) + 11a = \frac{x}{2a}$

$x_A, y_A < 0 \quad y' = 4ax - \omega = 1 \rightarrow 4ax_A = 1 \rightarrow x_A = \frac{1}{4a}$   
 $y_A = x_A$

$\rightarrow a^2 = \frac{9}{32} = \frac{1}{4} \frac{x_A^2}{x_A^2} \rightarrow a = \frac{1}{4} \rightarrow a = \frac{-1}{4}$  (175)

$a = \frac{1}{4} \rightarrow$  نقطه مثبت  $\rightarrow x^2 - 4x + 4 = (x-2)^2 = 0 \rightarrow$  نقطه سaddle

$y = x^3 - 12x + 2 \rightarrow y' = 3x^2 - 12 = 0 \rightarrow x = \pm 2$   
 $y'' = 6x \xrightarrow{x=-2} y'' = -12$  (Max)  
 $\xrightarrow{x=2} y'' = 12$  (min)

x	-2	2
y'	+ 0 -	- 0 +
y	↗	↘
	max	min

$\rightarrow \min = f(2) = -14$  ✓ (2)

$y = x^2 + ax^2 - 2bx - 2 \rightarrow y' = 2x^2 + 2ax - 2b$   
 $x=0 \rightarrow -2b=0 \Rightarrow b=0$   
 $x=-2 \rightarrow 12-4a=0 \rightarrow a=3$

$\Rightarrow y = x^2 + 2x^2 - 2$   
 $x_1=0 \rightarrow y_1=-2$   
 $x_2=-2 \rightarrow y_2=0$

$d_0 = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2} = \sqrt{(0+2)^2 + (-2-0)^2} = \sqrt{8} = 2\sqrt{2}$  ✓ (2)

$f(x) = x^2 - \omega|x| \xrightarrow{x>\omega} f(x) = x^2 - \omega x \rightarrow$    $\xrightarrow{x<\omega} f(x) = x^2 + \omega x$

$y = |f(x)| \xrightarrow{f(x)>0} \begin{cases} y = x^2 - \omega x; x > \omega \\ y = x^2 + \omega x; x < -\omega \end{cases}$

$\rightarrow m=1, n=3 \rightarrow \frac{n}{m} = \frac{3}{1}$  ✓ (2)

$f(x) < 0 \rightarrow \begin{cases} y = -x^2 + \omega x, 0 < x < \omega \rightarrow y' = -2x + \omega = 0 \rightarrow x = \frac{\omega}{2} \\ y = -x^2 - \omega x, -\omega < x < 0 \rightarrow y' = -2x - \omega = 0 \rightarrow x = -\frac{\omega}{2} \end{cases}$

x	-omega/2	omega/2
y	+ 0 -	- 0 +
	↗	↘
	max	min

x	-omega/2	omega/2
y	+ 0 -	- 0 +
	↗	↘
	max	min

$$y = |f(x)| = |2x+3| \xrightarrow{x > 0} |2^x+3| \rightarrow y' = \frac{(2^x+3)(\ln 2)}{2^x+3} = \ln 2 > 0 \rightarrow x = -\frac{3}{\ln 2} \quad |x > 0|$$

$$x < 0 \rightarrow |-2^x+3| \rightarrow y' = \frac{(-2^x+3)(-\ln 2)}{-2^x+3} = \ln 2 < 0 \rightarrow x = \frac{3}{\ln 2} \quad |x < 0|$$

مشقی  $\begin{cases} f'_-(0) = -3 \\ f'_+(0) = 3 \end{cases} \Rightarrow$  نقش در  $x=0$  ندارد  $\rightarrow$  نقطه برآیی است ✓ ② یک نقطه برآیی دارد.

$$y = \sqrt{x} |x-a| \rightarrow y = x^{\frac{1}{2}} |x-a| \xrightarrow{y'=0} x = \frac{(\frac{1}{2}x) + (1 \cdot x \cdot 0)}{\frac{1}{2} + 1} = \frac{a}{3}$$

$$y_{max} = 1 \Rightarrow f\left(\frac{a}{3}\right) = 1 \Rightarrow \sqrt{\frac{a}{3}} \cdot \left|\frac{a}{3} - a\right| = 1$$

$$\Rightarrow \sqrt{\frac{a}{3}} \cdot |a| = \frac{1}{3} \xrightarrow{a > 0} \frac{a^{\frac{3}{2}}}{\sqrt{3}} = \frac{1}{3} \rightarrow a^{\frac{3}{2}} = \frac{\sqrt{3}}{3} \rightarrow a = \left(\frac{\sqrt{3}}{3}\right)^{\frac{2}{3}} \rightarrow a = \frac{\sqrt{3}}{3}$$

②

$$f(x) = \sqrt{2|x|-x} \xrightarrow{x > 0} \sqrt{x^2-2} \xrightarrow{f'=0} x \in [1, +\infty) \Rightarrow D_f = [-1, 0] \cup [1, +\infty)$$

$$x < 0 \rightarrow \sqrt{-x^2-2} \xrightarrow{f'=0} x \in [-1, 0]$$

$$f(x) = \begin{cases} \sqrt{-x^2-2} & x < 0 \\ \sqrt{x^2-2} & x > 0 \end{cases} \rightarrow f'(x) = \frac{x-1}{2\sqrt{x^2-2}} = 0 \rightarrow x-1=0 \rightarrow x = \frac{1}{2} \quad |x > 0|$$

$x < 0 \rightarrow f'(x) = \frac{-x-1}{2\sqrt{-x^2-2}} = 0 \rightarrow -x-1=0 \rightarrow x = -\frac{1}{2} \quad |x < 0|$

$\begin{matrix} m=1 \\ n=0 \\ k=2 \end{matrix} \Rightarrow \frac{km+n}{k-n} = 1 \quad \checkmark$

$x$	$+$	$0$	$-$
$y$	$\nearrow$	$\searrow$	$\searrow$
$y$	$\nearrow$	$\searrow$	$\searrow$

$$y = \frac{mx+r}{x-l+m} \rightarrow y' = \frac{m(x-l+m) - (mx+r)}{(x-l+m)^2} = \frac{m^2 - m - r}{(x-l+m)^2} < 0 \Rightarrow (m-r)/(m+1) < 0$$

$$\rightarrow m \in (-1, 2) \rightarrow m = 0, 1 \rightarrow \text{مستقر} \quad \checkmark \quad \text{②}$$

$$f(x) = \frac{x}{1-2|x|} \xrightarrow{x > 0} \frac{x}{1-2^x} \rightarrow y' = \frac{1-2^x - (-2^x)}{(1-2^x)^2} = \frac{1+2^x}{(1-2^x)^2} = 0 \rightarrow 1-2^x = 0 \rightarrow x = 1 \quad \checkmark$$

$$x < 0 \rightarrow \frac{x}{1+2^x} \rightarrow y' = \frac{1+2^x - (2^x)}{(1+2^x)^2} = \frac{1-2^x}{(1+2^x)^2} = 0 \rightarrow 1-2^x = 0 \rightarrow x = -1 \quad \checkmark$$

② یک نقطه برآیی دارد ✓ نقطه برآیی