امین شهبازی 17.5

$$f(x)=1-\frac{a}{x} \quad ,[1,3]$$ -1

شیب متوسط: $\frac{f(b)-f(a)}{b-a} \Rightarrow \frac{f(3)-f(1)}{3-1} = \frac{1-\frac{a}{3}-(1-a)}{2} = \frac{\frac{2}{3}a}{2} = \boxed{\frac{a}{3}}$

2

آهنگ لحظه‌ای: $\frac{df(x)}{dx} = \frac{a}{x^2} \Rightarrow \frac{a}{x^2} = \frac{a}{3} \quad \frac{1}{x^2} = \frac{1}{3} \quad x^2 = 3 \quad x = \begin{cases} \sqrt{3} & \text{ق ق} \\ -\sqrt{3} & \text{غ ق ق} \end{cases}$

-2

$y_1 = 2ax^2 - 5x + 18a \xrightarrow{A} 2aA^2 - 5A + 18a = A$

$2aA^2 - 6A + 18a = 0 \quad aA^2 - 3A + 9a = 0 \longrightarrow \Delta = 0 \quad 9 - 4(9a^2) = 0$

$y_2 = x \quad y = A$

$4a^2 = 1 \quad a^2 = \frac{1}{4}$

$a = \pm\frac{1}{2}$

$y' = 4ax - 5 \quad 4ax - 5 = 1$

$y_2' = 1 \quad 4ax = 6 \quad aA = \frac{3}{2}$

2 $A<0$ چون با نیمساز ناحیه سوم است

$\boxed{a = -\frac{1}{2}} \quad \underline{a<0} \Longleftarrow aA>0$

-3

$y = x^3 - 12x + 2$

$y' = 3x^2 - 12 \quad 3(x^2-4) = 3(x-2)(x+2)$

x	$-\infty$		-2		2		$+\infty$
y'		+	0	−	0	+	
y		↗	18 max	↘	−14 min	↗	

$y_{-2} = (-2)^3 - 12(-2) + 2$

$-8 + 24 + 2 = 18 \quad y_2 = (2)^3 - 12(2) + 2 = 8 - 24 + 2 = -14$

2

$\boxed{-14 = y_{min}}$

-4

$y = x^3 + ax^2 - 2bx - 4 \quad y' = 3x^2 + 2ax - 2b \xrightarrow{x=0} \boxed{b=0}$

$x = \{0, 2\} \qquad \xrightarrow{x=2} 12 + 4a = 0 \quad \boxed{a = -3}$

$y = x^3 - 3x^2 - 4 \xrightarrow{x=0} y = -4$

$\xrightarrow{x=2} 8 - 12 - 4 = -8$

$x_2x_1 = \sqrt{(2-0)^2 + (\underbrace{-8-(-4)}_{-4})^2} = \sqrt{4+16} = \sqrt{20} = \boxed{2\sqrt{5}}$

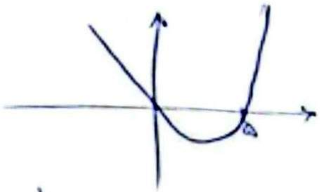
2

۵-

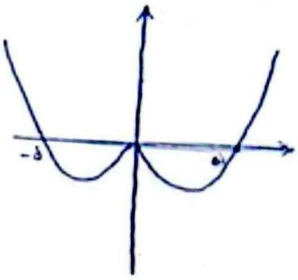
$f(x)=x^2-5|x|$

$|f(x)|=|x^2-5|x||$

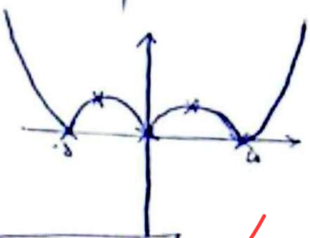
(۱) $f(x)=x(x-5)$



(۲) $f(x)=|x|(|x|-5)$



(۳) $f(x)=||x|(|x|-5)|$



۲

$m=2$

$n=3$

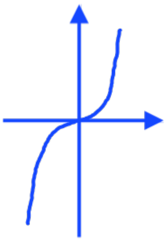
$\boxed{\frac{n}{m}=\frac{3}{2}}$

۶-

۰

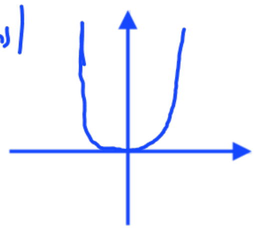
$f(x)=\begin{cases}x^3+3x & x\geq 0\\ -x^3+3x & x<0\end{cases}\rightarrow f'(x)=\begin{cases}3x^2+3 & x\geq 0\\ -3x^2+3 & x<0\end{cases}\rightarrow f'_+(0)=f'_-(0)=3$

$f(x)$



$\rightarrow$

$|f(x)|$



$\rightsquigarrow$ یک نقطه بحرانی $x=0$

۷-

$f(x)=\sqrt[3]{x^2}\,|x-a|$

$[0,a]$

$\max = 1.5$

$$f'(x)=\frac{2x|x-a|}{3\sqrt[3]{x}}+\frac{(x-a)\sqrt[3]{x^2}}{|x-a|}=\frac{2x(x-a)^2+3x(x-a)}{3\sqrt[3]{x}\,|x-a|}=\frac{x\cancel{|x-a|}\left(2(x-a)+3\right)}{3\sqrt[3]{x}\,\cancel{|x-a|}}=0$$

$x=0\ ,\ x-a=-\frac{3}{2}$

$f(0)=0$

$f(x)=1.5\qquad \sqrt[3]{x^2}\,\underset{|-\frac{3}{2}|}{|\cancel{x-a}|}=\frac{3}{2}\qquad \sqrt[3]{x^2}=1$

$x=1\qquad x-a=-\frac{3}{2}\qquad 1-a=-\frac{3}{2}\qquad \boxed{a=\frac{5}{2}}$

۲

۸-

$f(x) = \sqrt{x|x| - x}$ $D_f = [-1, 0] \cup [1, +\infty)$

$x > 0 = \sqrt{x^2 - x}$ $y' = \dfrac{2x - 1}{2\sqrt{x^2 - x}} = 0$ $2x - 1 = 0$

$x = 1/2$ غقق

$2\sqrt{x^2 - x} = 0$ $x = \{0, 1\}$

$x < 0 = \sqrt{-x^2 - x}$ $y' = \dfrac{-2x - 1}{2\sqrt{-x^2 - x}} = 0$ $-2x - 1 = 0$ $x = -1/2$ قق

$-x^2 - x = 0$ $x = \{0, -1\}$

$m \Rightarrow \underset{\text{نسبی}}{\max} = 1$

$n = \underset{\text{نسبی}}{\min} = 0$

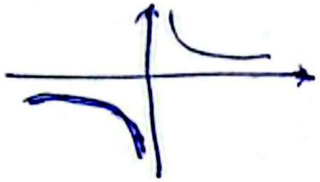
$k = 4$ ت ع

$\dfrac{km + n}{k - n} = \boxed{\dfrac{4 + 0}{4 - 0} = \dfrac{4}{4} = 1}$ ✓

۲

$-1<m<2 \,,\, m\neq 2 \to -1<m<2 \quad (I)$

$y = \frac{mx+2}{x-1+m}$ $\quad \frac{mx}{x} = 1$ $\quad m=1$



$\boxed{(1,+\infty)}$ $\quad m \neq 2$ $\quad$ 1,5

ریشه مخرج $= x = 1-m < 1 \to m \geq 0 \quad (II)$

$y' = \frac{m(m-1)-2}{(x+(m-1))^2}$ $\quad m(m-1)-2<0$ $\quad (m+1)(m-2)<0$

-1		2	
$-$	$-$		$+$

$(I)\cap(II) \to m=0,1$

$0<m<2$

$-1<m<2$

$0<m$ $\quad , -1<m<2 \longrightarrow \boxed{m=1} \leftarrow$ پیوسته

$f(x) = \frac{x}{1-x|x|}$ $\quad D_f = R-\{1\}$ $\quad$ 2

$x>0 \quad y = \frac{x}{1-x^2}$ $\quad y' = \frac{1-x^2-(-2x)x}{(1-x^2)^2} = \frac{1+x^2}{(1-x^2)^2}$ $\quad \frac{1+x^2}{(1-x^2)^2} = 0$ $\quad 1+x^2=0 \quad x^2=-1$ غ ق ق

$1-x^2=0 \quad x^2=1 \quad x=\pm 1$ $\to \boxed{-1 \text{ غ ق ق}}$, $\to +1$ غ ق ق

$x<0 \quad y = \frac{x}{1+x^2}$ $\quad y' = \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2} = 0$ $\quad 1-x^2=0 \quad x^2=\pm 1$ $\quad \boxed{x=-1}$

یک نقطه بحرانی $\boxed{x=-1}$