امین شهبازی

-۱

$f(x) = 1 - \frac{a}{x} \quad , [1, 3]$

آهنگ متوسط $= \frac{f(b) - f(a)}{b - a} \Rightarrow \frac{f(3) - f(1)}{3 - 1} = \frac{1 - \frac{a}{3} - (1 - a)}{2} = \frac{\frac{2}{3}a}{2} = \boxed{\frac{a}{3}}$

آهنگ لحظه‌ای $= \frac{df(x)}{dx} = \frac{a}{x^2} \Rightarrow \frac{a}{x^2} = \frac{a}{3} \quad \frac{1}{x^2} = \frac{1}{3} \quad x^2 = 3 \quad x = \begin{cases} \boxed{+3 \text{ ق ق}} \\ -3 \text{ غ ق ق} \end{cases}$

-۲

$y_1 = 2ax^2 - 5x + 18a \quad \xrightarrow{A} \quad 2aA^2 - 5A + 18a = A$

$2aA^2 - 6A + 18a = 0 \quad aA^2 - 3A + 9a = 0 \longrightarrow \Delta = 0 \quad 9 - 4(9a^2) = 0$

$y_2 = x \quad y = A \qquad 4a^2 = 1 \quad a^2 = \frac{1}{4}$

$a = \pm\frac{1}{2}$

$y' = 4ax - 5 \qquad 4ax - 5 = 1$

$y_2' = 1 \qquad 4ax = 6 \quad aA = \frac{3}{2}$

چون y نیمساز ناحیه سوم است $A < 0$

$\boxed{a = -\frac{1}{2}} \qquad \underline{a < 0} \Leftarrow aA > 0$

-۳

$y = x^3 - 12x + 2$

$y' = 3x^2 - 12 \quad 3(x^2 - 4) = 3(x - 2)(x + 2)$

x	$-\infty$		-2		2		$+\infty$
y'		+	0	−	0	+	
y		↗	18 max	↘	−14 min	↗	

$y_{-2} = \cancel{(-2)^3} - \cancel{12(-2)} + 2$
$= -8 + 24 + 2 = 18 \qquad y_2 = (2)^3 - 12(2) + 2 = 8 - 24 + 2 = -14$

$\boxed{\begin{vmatrix} 2 \\ -14 \end{vmatrix} = y_{min}}$

-۴

$y = x^3 + ax^2 - 2bx - 4 \qquad y' = 3x^2 + 2ax - 2b \xrightarrow{x=0} \boxed{b = 0}$

$x = \{0, 2\} \qquad \xrightarrow{x=2} 12 + 4a = 0 \quad \boxed{a = -3}$

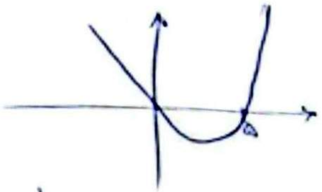
$y = x^3 - 3x^2 - 4 \quad \xrightarrow{x=0} y = -4 \qquad x_2x_1 = \sqrt{(2-0)^2 + (\underbrace{-8-(-4)}_{-4})^2} = \sqrt{4 + 16} = \sqrt{20} = \boxed{2\sqrt{5}}$

$\xrightarrow{x=2} 8 - 12 - 4 = -8$

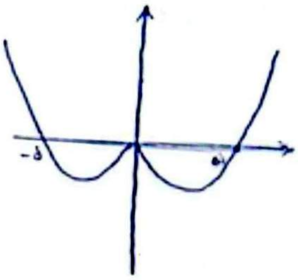
$$f(x)=x^2-5|x|$$ ۵-

$$|f(x)|=|x^2-5|x||$$

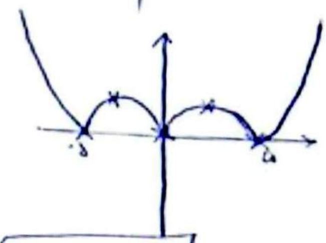
(۱) $f(x)=x(x-5)$



(۲) $f(x)=|x|(|x|-5)$



(۳) $f(x)=||x|(|x|-5)|$



$m=2$

$n=3$ $\boxed{\dfrac{n}{m}=\dfrac{3}{2}}$

$f(x)=\sqrt[3]{x^2}\,|x-a|$ ۷-

$[0,a]$

$\max = 1.5$

$$f'(x)=\frac{2x|x-a|}{3\sqrt[3]{x}}+\frac{(x-a)\sqrt[3]{x^2}}{|x-a|}=\frac{2x(x-a)^2+3x(x-a)}{3\sqrt[3]{x}\,|x-a|}=\frac{x\cancel{|x-a|}(2(x-a)+3)}{3\sqrt[3]{x}\,\cancel{|x-a|}}=0$$

$x=0\ ,\ x-a=-\frac{3}{2}$

$f(0)=0$

$f(x)=1.5 \quad \sqrt[3]{x^2}\,\underset{|-\frac{3}{2}|}{|x-a|}=\frac{3}{2} \quad \sqrt[3]{x^2}=1 \quad x=1$

$x-a=-\frac{3}{2}$

$1-a=-\frac{3}{2}$ $\boxed{a=5/2}$

$f(x) = \sqrt{x|x|-x}$ $\quad D_f = [-1, 0] \cup [1, +\infty)$

$x > 0 \Rightarrow \sqrt{x^2 - x}$ $\quad y' = \dfrac{2x-1}{2\sqrt{x^2-x}} = 0$ $\quad 2x - 1 = 0$

$x = 1/2$ غ ق ق

$2\sqrt{x^2 - x} = 0 \quad x = \{0, 1\}$

$x < 0 \Rightarrow \sqrt{-x^2 - x}$ $\quad y' = \dfrac{-2x-1}{2\sqrt{-x^2-x}} = 0$ $\quad -2x - 1 = 0 \quad x = -1/2$ ق ق

$-x^2 - x = 0 \quad x = \{0, -1\}$

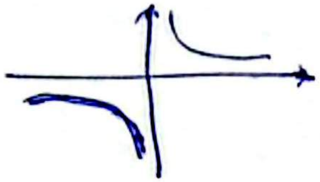
$m \Rightarrow \underset{\text{نسبی}}{max} = 1$

$n = \underset{\text{نسبی}}{min} = 0$

$k = 4$ ت ع

$\dfrac{km+n}{k-n} \Rightarrow \boxed{\dfrac{4+0}{4-0} = \dfrac{4}{4} = 1}$

$y = \dfrac{mx+2}{x-1+m}$ $\quad \dfrac{mx}{x} = 1$ $\quad m=1$



$\boxed{(1, +\infty)}$ $\quad m \neq 2$

$y' = \dfrac{m(m-1)-2}{(x+(m-1))^2}$ $\quad m(m-1)-2<0$ $\quad (m+1)(m-2)<0$

$-1 \quad 2$

$- \quad \circ \quad - \quad \circ \quad +$

$0<m<2$

$-1<m<2$

$\cancel{\ }$ $0<m$, $-1<m<2 \Longrightarrow \cancel{1<m<2}$ $\boxed{m=1} \rightarrow$ یک مقدار

$f(x) = \dfrac{x}{1-x|x|}$ $\quad D_f = R - \{1\}$

$x>0 \quad y = \dfrac{x}{1-x^2}$ $\quad y' = \dfrac{1-x^2-(-2x)x}{(1-x^2)^2} = \dfrac{1+x^2}{(1-x^2)^2}$ $\quad \dfrac{1+x^2}{(1-x^2)^2} = 0$ $\quad 1+x^2=0 \quad x^2=-1$ غ ق ق

$1-x^2=0 \quad x^2=1 \quad x=\pm 1$ $\rightarrow \boxed{-1 \text{ ق ق}}$

$\rightarrow +1$ غ ق ق

$x<0 \quad y = \dfrac{x}{1+x^2}$ $\quad y' = \dfrac{1+x^2-2x^2}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2} = 0$ $\quad 1-x^2=0 \quad x^2=\pm 1$ $\boxed{x=-1}$

یک نقطه بحرانی $\boxed{x=-1}$