۱۴ آزمون نوری

۱-

$f(1) = 1-a$

$f(2) = 1-\frac{a}{2}$

$\rightarrow m = \frac{1-\frac{a}{2}-1+a}{2-1} = \frac{\frac{a}{2}}{1} = \frac{a}{2}$

$f'(x) = \frac{+a}{x^2} \rightarrow \frac{a}{x^2} = \frac{a}{2}$

$x \begin{cases} \sqrt{2} \\ -\sqrt{2} \end{cases}$ غ ق ق

$x = \sqrt{2}$ ق ق ق

(۲)

۲-

$4ax - 5 = 1 \rightarrow 4ax = 6 \quad ax = \frac{3}{2}$

$2ax^2 - 5x + 18a = x$

$2ax^2 - 6x + 18a = 0$

$3x - 6x + 18a = 0 \rightarrow 18a = 3x \quad x = 6a$

$6a^2 = \frac{3}{2} \quad a^2 = \frac{1}{4}$

$a \begin{cases} \frac{1}{2} \text{ غ ق ق} \\ -\frac{1}{2} \checkmark \end{cases}$

$a = -\frac{1}{2}$ ق ق ق

(۲)

۳-

$y = x^3 - 12x + 1 \rightarrow y' = 3x^2 - 12$

$3x^2 - 12 = 0 \quad 3x^2 = 12 \quad x \subset \begin{matrix} 2 \\ -2 \end{matrix}$

	-2		2	
y'	+	−	+	
	↗	↘	↗	

$\min = (2, -15)$ نقطه

$= -15$ ق ق ق

(۲)

۴-

$y = x^3 + ax^2 - 2bx - 4 \rightarrow y' = 3x^2 + 2ax - 2b$

$-2b = 0 \rightarrow b = 0 \quad 12 - 4a = 0 \rightarrow a = 3 \Rightarrow y = x^3 + 3x^2 - 4$

$\begin{vmatrix} 0 & -2 \\ -4 & 0 \end{vmatrix}$

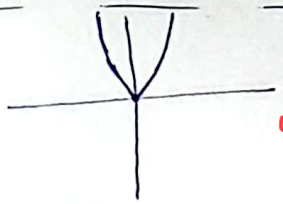
$\sqrt{(0+2)^2 + (-4)^2} = \sqrt{4+16} = 2\sqrt{5}$ ق ق ق

(۲)

۶-

$y = x(|x| + 2)$

$-x^2 + 2x \mid x^2 + 2x$



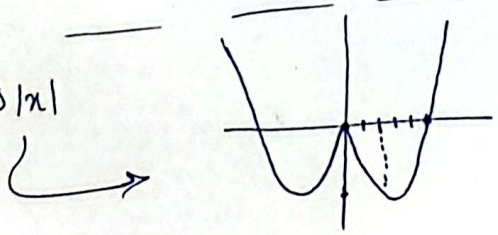
یک نقطه ای

بحرانی

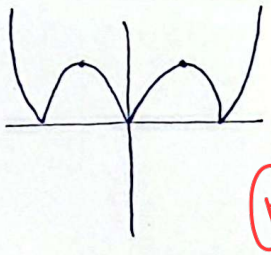
(۲)

۵-

$x^2 - 5|x|$



$|x^2 - 5|x||$



$m = 2$

$n = 3$

$\frac{n}{m} = \frac{3}{2}$ ق ق ق

(۲)

۷-

$x \in [0,a] \rightarrow |x-a| = -(x-a) \rightarrow f(x) = -\sqrt[3]{x^2}(x-a) = -x^{\frac{5}{3}} + ax^{\frac{2}{3}}$

$f'(x) = -\frac{5}{3}x^{\frac{2}{3}} + \frac{2}{3}ax^{-\frac{1}{3}} = 0 \rightarrow \frac{1}{3}x^{-\frac{1}{3}}(-5x+2a) = 0 \rightarrow \begin{cases} x=0 \\ x=\frac{2}{5}a \rightarrow max \checkmark \end{cases}$

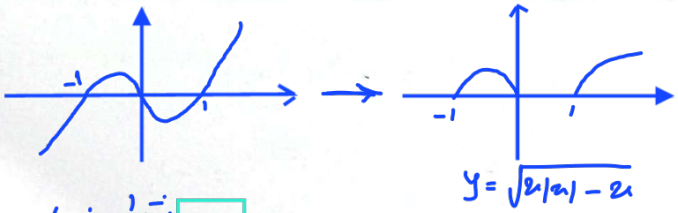
$f(x_{max}) = 1,5 \rightarrow f(\frac{2}{5}a) = \frac{3}{2} \rightarrow -\sqrt[3]{\frac{4}{125}a^2}\left(\frac{2}{5}a - a\right) = \frac{3}{2} \rightarrow ax\sqrt[3]{\frac{4}{125}a^2} = \frac{5}{2}$

توان ۳ $\rightarrow a^3 \times \frac{4}{125}a^2 = \frac{125}{8} \rightarrow a^5 = \frac{125}{8} \times \frac{32}{4} = (\frac{5}{2})^5 \rightarrow a = \frac{5}{2} = 2,5$

۵

۸-

$y = |x|x - x \rightarrow \begin{cases} x^2 - x & x \geq 0 \quad (I) \\ -x^2 - x & x < 0 \quad (II) \end{cases}$



(تعداد مجانب) $K=4$ ، (max تابع) $M=1$ ، (min تابع) $n=0$

$\frac{Km+n}{k-n} = \frac{4\times 1 - 0}{4-0} = \frac{4}{4} = 1$

۵

۹-

$y' = \frac{m(x-1+m) - mx - 2}{(x-1+m)^2} = \frac{mx - m + m^2 - mx - 2}{(x-1+m)^2} = \frac{m^2 - m - 2}{(x-1+m)^2}$

همواره (+)

$m^2 - m - 2 = (m-2)(m+1)$ (I)

	-1		2	
	+	‖	-	+

$1 - m \leq 1 \rightarrow m \geq 0$ (II)

$(I) \cap (II) = [0,2)$

$m = 0,1$ ✓

۲

۱۰-

$y = \frac{x}{1-x|x|} \rightarrow D_f = R - \{1\}$

		0		
$\frac{x}{1+x^2}$		0		$\frac{x}{1-x^2}$
$\frac{1+x^2-2x^2}{(1+x^2)^2}$				$\frac{1-x^2+2x^2}{(1-x^2)^2}$
$\frac{1-x^2}{(1+x^2)^2}$				$\frac{x^2+1}{(1-x^2)^2}$
نزولی		غ ن ا		صعودی
		-1 ✓		
صعودی		غ ن ا		نزولی
				-1 غ ن ا

$y \rightarrow$

یک نقطه‌ی بحرانی دارد ✓

۲