

$f(1) = 1 - a$ $\frac{(1 - \frac{a}{3}) - (1 - a)}{3 - 1} = \frac{a}{3}$

$f(3) = 1 - \frac{a}{3}$

$f'(x) = \frac{a}{x^2} = \frac{a}{3} \rightarrow x = \pm\sqrt{3}$

$\begin{cases} x = -\sqrt{3} \times \\ x = \sqrt{3} \checkmark \end{cases}$

(۱, ۱/۳)

$f(x) = x \rightarrow f'(x) = 1$

$y' = 3ax - 2$

$3ax^2 - 2x + 11a = x \rightarrow 3ax^2 - 3x + 11a = 0 \rightarrow \Delta = 9 - 4(3a)(11a) = 0$

$3ax - 2 = 1 \rightarrow x = \frac{3}{3a} \xrightarrow{x < 0} a = -\frac{1}{3}$

$a = \pm \frac{1}{3}$

(۲)

$y = x^3 - 12x + 2$

$y' = 3x^2 - 12$

$\min \rightarrow \begin{cases} f'(x) = 0 \\ f''(x) = - \\ f''(x) = + \end{cases} \rightarrow \begin{matrix} 2 \\ -12 \end{matrix}$

x	$-\infty$	-2	2	$+\infty$	
y'	+	0	-	0	+
y		↗	↘	↗	
		11	-14		

(۳)

$y = x^3 + ax^2 - 2bx - 4 \rightarrow y = x^3 + 3x^2 - 4$

$y' = 3x^2 + 2ax - 2b \rightarrow y' = 3x^2 + 6x$

$x = 0 \rightarrow -2b = 0 \rightarrow b = 0$

$x = -2 \rightarrow 12 - 4a - 2b = 0 \xrightarrow{b=0} 12 - 4a = 0 \rightarrow a = 3$

$d = \sqrt{(0+2)^2 + (-4-0)^2} = 2\sqrt{5}$

x	$-\infty$	-2	0	$+\infty$	
y'	+	0	-	0	+
y		↗	↘	↗	
		0	-4		

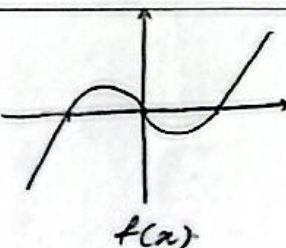
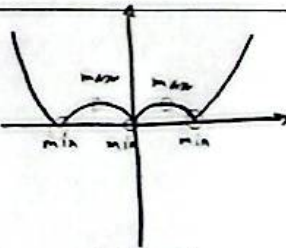
(۴)

$f(x) \begin{cases} x \geq 0 & x^2 - 2x \\ x < 0 & x^2 + 2x \end{cases}$

$m = 2$

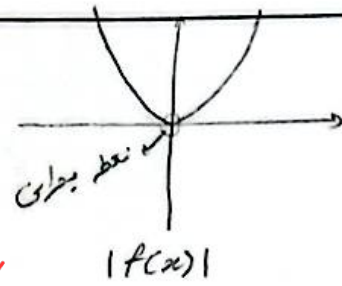
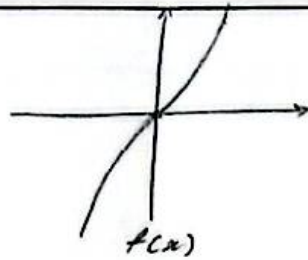
$n = 3$

$\frac{n}{m} = \frac{3}{2}$

(۵)

$$f(x) \begin{cases} x \geq 0 & x^2 + 3x \\ x < 0 & -x^2 + 3x \end{cases}$$



تعداد نقاط برای: 1

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$$0 \leq x \leq a \rightarrow f(x) = -\sqrt[3]{x^2} (x-a)$$

$$f'(x) = -\frac{2x}{\sqrt[3]{x^2}} (x-a) - \sqrt[3]{x^2} = \frac{ax - 2x^2}{\sqrt[3]{x^2}} = \frac{x(a-2x)}{x\sqrt[3]{x}} = \frac{a-2x}{\sqrt[3]{x}}$$

$$x | -\infty \quad \frac{a}{2} \quad a \quad +\infty$$

y'	-	+	0	-	+
y	↘	↗	↘	↗	↗

$$-\frac{\sqrt[3]{ax}}{x} \times (-\frac{a}{2}) = \frac{a}{2} \rightarrow \frac{a \times \sqrt[3]{ax}}{x} = \frac{a}{2} \rightarrow a \times \sqrt[3]{ax} = \frac{a}{2}$$

$$\rightarrow a = \sqrt[3]{\frac{a}{2}}$$

$$f(x_{max}) = \frac{1}{2}a \rightarrow f(\frac{1}{2}a) = \frac{1}{2} \rightarrow -\sqrt[3]{\frac{1}{8}a^3} (\frac{1}{2}a - a) = \frac{1}{2} \rightarrow a \times \sqrt[3]{\frac{1}{8}a^3} = \frac{1}{2}$$

$$\frac{1}{2}a \rightarrow a^2 \times \frac{1}{8}a^3 = \frac{1}{2}a \rightarrow a^5 = \frac{1}{2}a \times \frac{2}{1} = \frac{1}{1} \rightarrow a = \frac{1}{2}$$

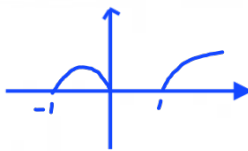
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$$f(x) \begin{cases} x > 0 & \sqrt{x^2-2x} \\ x < 0 & \sqrt{-x^2-x} \end{cases}$$

$$\rightarrow f'(x)$$

$$\begin{cases} x \geq 0 & \frac{2x-1}{2\sqrt{x^2-2x}} \\ x < 0 & \frac{-2x-1}{2\sqrt{-x^2-x}} \end{cases}$$



1/2

$$D_f = [-1, 0] \cup [1, +\infty)$$

x	-1	-1/2	0	1	
y'	+	0	-	0	+
y	↗	↘	↘	↗	↗

max $x \rightarrow (-\frac{1}{2}, \frac{1}{2})$
min $x \rightarrow$ ندارد

$$m=1 \quad y = \sqrt{2|x|-2x}$$

K=2

برای $\rightarrow (-1, 0) \quad (0, 0) \quad (1, 0) \quad k=2$

$$\frac{km+n}{k-n} = \frac{2 \times 1 + 0}{2-0} = 1$$

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$$y = \frac{mx+1}{x+m-1}$$

$$y' = \frac{m^2 - m - 1}{(x+m-1)^2} = \frac{(m-2)(m+1)}{(x+m-1)^2} \quad |x| \rightarrow f'(x) < 0 \rightarrow -1 < m < 2$$

x	$-\infty$	$+\infty$
y'	-	-
y	↘	↘

$m \in \{0, 1\}$ دو مقدار صحیح

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$$f(x) \begin{cases} x \geq 0 & \frac{x}{1-x^2} \\ x < 0 & \frac{x}{1+x^2} \end{cases}$$

$$\rightarrow f'(x)$$

$$\begin{cases} \frac{1+x^2}{(1-x^2)^2} & x \geq 0 \\ \frac{1-x^2}{(1+x^2)^2} & x < 0 \end{cases}$$

$$D_f = \mathbb{R} - \{1\}$$

x	-1	1
y'	-	+
y	↘	↗

یک نقطه برای

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