نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۵ کلاس .....

**۱**

$f(1) = 1 - a$

$f(3) = 1 - \frac{a}{3}$

$\frac{(1-\frac{a}{3}) - (1-a)}{3-1} = \frac{a}{3}$

$f'(x) = \frac{a}{x^2} = \frac{a}{3} \longrightarrow \boxed{x = \pm\sqrt{3}}$

**۲**

$f(x) = x \longrightarrow f'(x) = 1$

$y' = 4ax - 5$

$2ax^2 - 5x + 18a = x \longrightarrow 2ax^2 - 6x + 18a = 0 \longrightarrow \Delta = 36 - 4(2a)(18a) = 0$

$a = \pm\frac{1}{2}$

$4ax - 5 = 1 \longrightarrow x = \frac{3}{2a} \xrightarrow{x<0} \boxed{a = -\frac{1}{2}}$

**۳**

$y = x^3 - 12x + 2$

$y' = 3x^2 - 12$

| $x$ | $-\infty$ | | $-2$ | | $2$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $y'$ | | $+$ | $0$ | $-$ | $0$ | $+$ | |
| $y$ | | ↗ | $18$ | ↘ | $-14$ | ↗ | |

$\min$ نسبی $\longrightarrow \begin{cases} f'(x) = 0 \\ f'_-(x) = - \\ f'_+(x) = + \end{cases} \longrightarrow \boxed{\begin{matrix} 2 \\ -14 \end{matrix}}$

**۴**

$y = x^3 + ax^2 - 2bx - 4 \longrightarrow y = x^3 + 3x^2 - 4$

$y' = 3x^2 + 2ax - 2b \longrightarrow y' = 3x^2 + 6x$

$x = 0 \longrightarrow -2b = 0 \longrightarrow b = 0$

$x = -2 \longrightarrow 12 - 4a - 2b = 0 \xrightarrow{b=0} 12 - 4a = 0 \longrightarrow a = 3$

| $x$ | $-\infty$ | | $-2$ | | $0$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $y'$ | | $+$ | $0$ | $-$ | $0$ | $+$ | |
| $y$ | | ↗ | | ↘ | | ↗ | |

$d = \sqrt{(0+2)^2 + (-4-0)^2} = \boxed{2\sqrt{5}}$

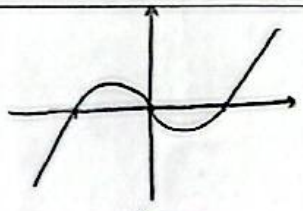
**۵**

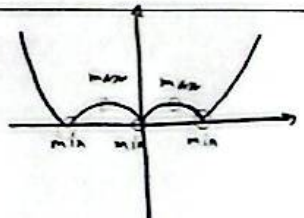
$f(x) \begin{cases} x \geq 0 & x^2 - 5x \\ x < 0 & x^2 + 5x \end{cases}$

$m = 2$

$n = 3$

$\frac{n}{m} = \frac{3}{2}$

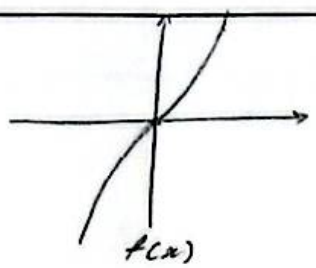


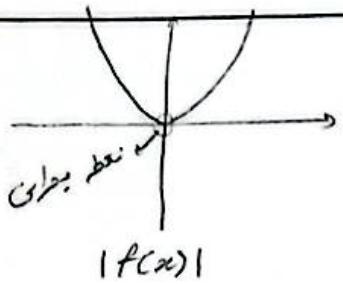




$|f(x)|$

۶

$$f(x)\begin{cases} x\geq 0 & x^2+3x \\ x<0 & -x^2+3x \end{cases}$$




تعداد نقطه بحرانی : ۱

۷

$$0\leq x\leq a \rightarrow f(x)=-\sqrt[3]{x^2}(x-a)$$

$$f'(x)=-\frac{2x}{2\sqrt[3]{x^4}}(x-a)-\sqrt[3]{x^2}=\frac{ax-2x^2}{\sqrt[3]{x^4}}=\frac{x(a-2x)}{x\sqrt[3]{x}}$$

| $x$ | $-\infty$ | $0$ | $\frac{a}{2}$ | $a$ | $+\infty$ |
|---|---|---|---|---|---|
| $y'$ | $-$ | $0$ | $+$ | $0$ | $-$ | $+$ |
| $y$ | ↘ | min | ↗ | max | ↘ | ↗ |

$$-\sqrt[3]{\frac{a^2}{4}}\times\left(-\frac{a}{2}\right)=\frac{3}{2}\rightarrow\frac{a\times\sqrt[3]{a^2}}{4}=\frac{3}{2}\rightarrow a\times\sqrt[3]{a^2}=6$$

$$\rightarrow a=\sqrt[5]{216}$$

۸

$$f(x)\begin{cases} x\geq 0 & \sqrt{x^2-x} \\ x<0 & \sqrt{-x^2-x} \end{cases}\rightarrow f'(x)\begin{cases} x\geq 0 & \frac{2x-1}{2\sqrt{x^2-x}} \\ x<0 & \frac{-2x-1}{2\sqrt{-x^2-x}} \end{cases}$$

$$D_f=[-1,0]\cup[1,+\infty)$$

| $x$ | $-1$ | $-\frac{1}{2}$ | $0$ | $1$ |
|---|---|---|---|---|
| $y'$ | $0$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $y$ | min | ↗ | max | ↘ | min | ↗ |

نسبی max → $(-\frac{1}{2},\frac{1}{2})$ $\quad m=1$

نسبی min → ندارد $\quad n=0$

بحرانی → $(-1,0)$ $(0,0)$ $(1,0)$ $\quad k=3$

$$\frac{km+n}{k-n}=\frac{3\times 1+0}{3-0}=1$$

۹

$$y=\frac{mx+2}{x+m-1}$$

$$y'=\frac{m^2-m-2}{(x+m-1)^2}=\frac{(m-2)(m+1)}{(x+m-1)^2}\xrightarrow{1<x} f'(x)<0\rightarrow -1<m<2$$

| $x$ | $-\infty$ | $1$ | $+\infty$ |
|---|---|---|---|
| $y'$ | $-$ | $\|$ | $-$ |
| $y$ | ↘ | $\|$ | ↘ |

$m\in\{0,1\}$ دو مقدار صحیح

۱۰

$$f(x)\begin{cases} x\geq 0 & \frac{x}{1-x^2} \\ x<0 & \frac{x}{1+x^2} \end{cases}\rightarrow f'(x)\begin{cases} x\geq 0 & \frac{1+x^2}{(1-x^2)^2} \\ x<0 & \frac{1-x^2}{(1+x^2)^2} \end{cases}$$

$$D_f=\mathbb{R}-\{1\}$$

| $x$ | $-1$ | $1$ |
|---|---|---|
| $y'$ | $-$ | $0$ | $+$ | $\|$ | $+$ |
| $y$ | ↘ | min | ↗ | $\|$ | ↗ |

یک نقطه بحرانی