

$f(x) = \sqrt{x-2|x|}$ $x \geq 0 \rightarrow f(x) = \sqrt{x-2x}$ $x < 0 \rightarrow f(x) = \sqrt{x+2x}$

$f'(x) = \frac{2x+1}{2\sqrt{x+2x}} = 0 \rightarrow x = -\frac{1}{2}$ (نقطه بحرانی)

$f'(x) = \frac{-2x+1}{2\sqrt{x-2x}} = 0 \rightarrow x = \frac{1}{2}$ (نقطه بحرانی)

جدول علامت: $x \mid \frac{1}{2} \mid$
 $y' \mid + \quad - \mid$
 $y \mid \nearrow \quad \searrow \mid$

گراف:

$k=f, m=1, n=0 \Rightarrow k+m+n=5$

$f(x) = \sqrt{x} + \sqrt{a-x}$ $\rightarrow f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{a-x}} = 0 \rightarrow \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{a-x}} \rightarrow a-x = x \rightarrow a=2x \rightarrow x = \frac{a}{2}$

$a-2x > 0 \rightarrow x < \frac{a}{2}$ $\Rightarrow D_f = [0, \frac{a}{2}]$

$f(0) = \sqrt{a}$
 $f(\frac{a}{2}) = \sqrt{\frac{a}{2}} \mid \text{min} \mid$
 $f(\frac{a}{2}) = \sqrt{\frac{a}{2}} + \sqrt{\frac{a}{2}} = \frac{1}{\sqrt{2}}(\sqrt{a}) + \frac{1}{\sqrt{2}}(\sqrt{a}) = \frac{2}{\sqrt{2}}(\sqrt{a}) \mid \text{max} \mid$

$\min \times \max = \sqrt{a} \rightarrow \frac{\sqrt{a}}{\sqrt{2}} \times \frac{2}{\sqrt{2}}\sqrt{a} = \sqrt{a} \rightarrow \frac{2}{2}a = \sqrt{a} \rightarrow 2a = \sqrt{a} \rightarrow a = \frac{1}{4}$

$\Rightarrow [a] = \frac{1}{4}$

$f(x) = \frac{x^2}{x^2-1} \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$

$f(x) = \begin{cases} \frac{x^2(x^2-1)}{x^2-1} & x > 1, x < -1 \\ -\frac{x^2(x^2-1)}{x^2-1} & -1 < x < 1 \end{cases} \Rightarrow f'(x) = \begin{cases} \frac{(x^2-1)(2x) - (x^2)(2x)}{(x^2-1)^2} & x > 1, x < -1 \\ -\frac{[(x^2-1)(2x) - (x^2)(2x)]}{(x^2-1)^2} & -1 < x < 1 \end{cases}$

$f'(x) = 0 \rightarrow 2x^3 - 2x^2 - 2x + 2 = 0 \rightarrow x^3 - x^2 - x + 1 = 0 \rightarrow x^2(x-1) - (x-1) = 0 \rightarrow (x^2-1)(x-1) = 0 \rightarrow x = 0$

$y = ax^2 + bx + c \Rightarrow y' = 2ax + b = 0 \xrightarrow{x=0} y' = c = 0$

$A(0,0) \Rightarrow d=0 \Rightarrow y = ax^2 + bx \xrightarrow{B(1,1)} 1 = a + b \Rightarrow a = 1 - b$

$y' = 2ax + b \xrightarrow{x=1} 2a + b = 0 \Rightarrow 2(1-b) + b = 0 \Rightarrow b = 2, a = 1 - 2 = -1$

$\Rightarrow a \cdot b = -2 \times 2 = -4$

$f(x) = 2|x-x^2| \quad x-x^2=0 \Rightarrow x = \pm 1$

$f(x) = -x^2 + 2x, \quad -\frac{1}{2} < x < \frac{3}{2}$

$f'(x) = -2x + 2 = 0 \Rightarrow x = 1$

جدول علامت: $x \mid -\frac{1}{2} \mid 1 \mid \frac{3}{2} \mid$
 $y' \mid - \quad 0 \quad + \quad 0 \quad - \mid$
 $y \mid \nearrow \quad \searrow \mid$

گراف:

$\min = -2$

$$y = x^2 + 2ax + b \quad \frac{dy}{dx} = 2x + 2a \rightarrow y' = -2x + 4a$$

$$x = -1 \quad y' = -2 - 4a = 0 \rightarrow a = -\frac{1}{2}$$

$$\rightarrow y = -x^2 - \frac{1}{2}x + b \quad A(-1, 1) \rightarrow 1 = -(-1)^2 - \frac{1}{2}(-1) + b \rightarrow b = \frac{3}{2}$$

$$\rightarrow \frac{b}{a} = \frac{\frac{3}{2}}{-\frac{1}{2}} = -3$$

(۲)

$$y = \frac{3}{2}x^2 + x + \frac{1}{2} \rightarrow y' = 3x + 1 = 0 \rightarrow x = -\frac{1}{3} \quad y = \frac{5}{6} \quad \text{min}$$

$$y = \frac{(ax+3)}{(a+1)x+(a-1)}$$

$$\text{مجاوبه} = (a+1)x + (a-1) = 0 \rightarrow x = \frac{-a+1}{a+1}$$

$$\text{مجاوبه افقی} : y = \frac{a}{2} = \frac{a}{a+1}$$

$$\frac{-a+1}{a+1} = -\frac{1}{3} \quad \& \quad \frac{a}{a+1} = \frac{5}{6} \Rightarrow a = 2$$

$$y = \frac{(2x+3)}{3x+1} = 0 \rightarrow 2x+3=0 \rightarrow x = -\frac{3}{2}$$

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$$y = \frac{bx^2+7}{3x^2+ax+1}$$

$$\text{مجاوبه} : 3x^2+ax+1=0 \quad x = -\frac{1}{3} \quad 3(-\frac{1}{3})^2 + a(-\frac{1}{3}) + 1 = 0 \rightarrow a = 2$$

$$\text{مجاوبه افقی} : \frac{b}{3} = 3 \rightarrow b = 9 \Rightarrow \frac{b}{a} = \frac{9}{2} = 4.5$$

(۲)

$$f(x) = \frac{x^2}{x^2-1} \quad x^2-1=0 \rightarrow x = \pm 1$$

$$y' = \frac{(2x(x^2-1) - x^2(2x))}{(x^2-1)^2} = \frac{2x^3 - 2x^3 - 2x^3}{(x^2-1)^2} = \frac{-2x^3}{(x^2-1)^2}$$

$$= \frac{-2x^3}{(x^2-1)^2} = 0 \rightarrow -2x^3 = 0 \rightarrow x = 0$$

x	0	1	2	3	4
y'	+	0	-	0	+
y	↗	↘	↗	↘	↗

تابع در بازه های $(0, 1)$ و $(2, 3)$ کاهنده است
بازه کاهنده $(2, 3)$ است که طول آن برابر با 1 است

(۲)

$$f(x) = \frac{x^2-3}{x^2-2} \quad D_f = \mathbb{R} - \{\pm\sqrt{2}\} \quad \text{حداکثر بازه شیب منفی است} \quad f'(x) = \frac{2x(x^2-2) - (x^2-3)(2x)}{(x^2-2)^2}$$

$$= \frac{2x^3 - 4x^2 + 6x - 2x^3 + 4x^2 - 6x}{(x^2-2)^2} = \frac{-2x^3 + 4x^2 - 6x}{(x^2-2)^2} = \frac{-2x(x^2 - 2x + 3)}{(x^2-2)^2}$$

x	$-\sqrt{3+2\sqrt{2}}$	$-\sqrt{2}$	$-\sqrt{3-2\sqrt{2}}$	0	$\sqrt{3-2\sqrt{2}}$	$\sqrt{2}$	$\sqrt{3+2\sqrt{2}}$
f'(x)	-	0	+	0	-	0	+

الگوریتم $f'(x) < 0 \Rightarrow x \in (-\sqrt{2}, \sqrt{2})$
بازه $(-\sqrt{2}, \sqrt{2})$

(۱, ۵)