

$f(x) = \begin{cases} \sqrt{x(1-x)} & 0 \leq x \leq 1 \\ \sqrt{x(1+x)} & x \leq -1 \end{cases} \Rightarrow f'(x) = \begin{cases} \frac{-2x+1}{2\sqrt{x(1-x)}} & 0 < x < 1 \\ \frac{2x+1}{2\sqrt{x(1+x)}} & x < -1 \end{cases}$

$f'(x) = 0 \Rightarrow x = \frac{1}{2}$  ✓  
 $f'$  ناموجود  $\Rightarrow -x(1-x) = 0 \Rightarrow x=0$  یا  $x=1$  ✓  
 $f'(x) = 0 \Rightarrow x = -\frac{1}{2}$  ✓  
 $f'$  ناموجود  $\Rightarrow x^2+x=0 \Rightarrow x=0$  یا  $x=-1$  ✓

$K = \{0, 1, -1\} = 3 \Rightarrow$  مشتق نشود  $\Rightarrow x=0$  یا  $x=-1$

$m+n+k = 3$   
 $m = \{ \frac{1}{2} \} = 1$   
 $n = 0$

$Df \rightarrow x \geq a \Rightarrow f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{a-x}} = 0 \Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{a-x}}$

$a-x = x \Rightarrow a = 2x \Rightarrow x = \frac{a}{2} \Rightarrow$  ماکزیمم  $\Rightarrow \frac{a}{2} \Rightarrow f(\frac{a}{2}) = \sqrt{\frac{a}{2}}$   
 مینیمم  $\Rightarrow \frac{a}{2} \Rightarrow f(\frac{a}{2}) = \sqrt{\frac{a}{2}}$

$\Rightarrow \sqrt{\frac{a}{2}} = \sqrt{\frac{a}{2}} = \sqrt{2} \Rightarrow a = 4$

$f(x) = \begin{cases} (x-\frac{1}{2})(x^2-4) = \frac{x^3-x^2-2x+2}{x^2-1} & |x| \geq 2 \\ (x-\frac{1}{2})(-x^2+4) = \frac{-x^3+x^2+2x-2}{x^2-1} & |x| < 2 \end{cases} \Rightarrow f'(x) = \begin{cases} \frac{(x^2-2x)(x^2-1) - (x^3-x^2-2x+2)(2x)}{(x^2-1)^2} & |x| \geq 2 \\ \frac{(x^2-2x)(x^2-1) - (-x^3+x^2+2x-2)(2x)}{(x^2-1)^2} & |x| < 2 \end{cases}$

$\Rightarrow f'(x) = 0 \Rightarrow x = 0$

جدول علامتی:
 

x	-2	-1	0	1	2
f'	-	+	+	-	-
f	↘	↗	↗	↘	↘

$f(x) = 0 \Rightarrow d = 0 \Rightarrow f(1) = a+b+c = 1$  |  $f'(0) = f'(1) = 0 \Rightarrow c = 0 \Rightarrow ax^2 + bx$

$\Rightarrow 2ax^2 + bx = A(x^2-x) \Rightarrow A = 2a \Rightarrow 2ax^2 - 2ax \Rightarrow 2b = -2a \Rightarrow a = -\frac{1}{2}b$

$\Rightarrow a+b = 1 \Rightarrow -\frac{1}{2}b + b = \frac{1}{2}b = 1 \Rightarrow b = 2$  و  $a = -1 \Rightarrow ab = -2$

$f(x) = \begin{cases} x^3-3x & |x| \geq \sqrt{3} \\ -2x^2+3x & |x| < \sqrt{3} \end{cases} \rightarrow f'(x) = \begin{cases} 3x^2-3 & |x| \geq \sqrt{3} \\ -4x+3 & |x| < \sqrt{3} \end{cases}$

$f'(x) = 0 \Rightarrow x = \pm 1$  ✓  
 $f'(x) = 0 \Rightarrow x = \pm 1$  ✓

$x = \pm \sqrt{3}$  بهرانی  $\rightarrow$  ریشه در صطلق  $\rightarrow$

جدول علامتی:
 

x	-\sqrt{3}	-1	1	\sqrt{3}
f'	+	-	+	-
f	↗	↘	↗	↘

طبق جدول تغییرات در بازه‌ی اطلاعاتی نقطه  $x = -1$  است  $\rightarrow$   $\text{Min} \{-1\}$

$f(x) = -x^3 + 3ax + b \rightarrow f(1) = 1 \Rightarrow 3a + b = 0 \Rightarrow 3a = -b$   
 (مشتق)  
 $\Rightarrow f(x) = -x^3 - bx + b \Rightarrow f'(x) = -3x^2 - b \Rightarrow f'(1) = 0$   
 $\Rightarrow -3 - b = 0 \Rightarrow b = -3 \Rightarrow a = 1 \Rightarrow \frac{b}{a} = \frac{-3}{1} = \boxed{-3}$

$f(x) = 3x + 1 \Rightarrow \min = \frac{-1}{3} \Rightarrow \frac{a}{a+1} = \frac{1}{3} \Rightarrow 3a = a+1$   
 $\Rightarrow 2a = 1 \Rightarrow a = \frac{1}{2}$   
 $\Rightarrow y = \frac{3x + 1}{3x + 1} = 0 \Rightarrow 3x + 1 = 0 \Rightarrow x = \frac{-1}{3}$

$f'(x) = \frac{f(x)(x^2-1) - x^2(f'(x))}{(x^2-1)^2} = \frac{x^4 - 3x^2}{(x^2-1)^2} \leq 0 \Rightarrow x^2 \leq 3x^2 \leq 0$   
 $\Rightarrow x = 2$  مشتق ناپذیر و با فرض مثبت  $\Rightarrow [0, 2) \cup (2, \sqrt{32})$   
 $\sqrt{32} - 2 = 2\sqrt{4} - 2 = 2(\sqrt{4} - 1) < 2 \Rightarrow 2(\sqrt{4} - 1)$

$f \Rightarrow y = 3 \Rightarrow \frac{b}{a} = 3 \Rightarrow \boxed{b = 12}$   
 $f \Rightarrow x = \frac{1}{2} \Rightarrow f(x) + a(x+1) = A(x+\frac{1}{2})^2 \Rightarrow A = 4$   
 $\Rightarrow f(x) + a(x+\frac{1}{2}) = 4(x+\frac{1}{2})^2 \Rightarrow \boxed{a = 4} \Rightarrow \frac{b}{a} = 3$

$f(x) < 0 \Rightarrow f(x) = \frac{x^4 - 3}{x^2 + 1} \Rightarrow f'(x) = \frac{4x^3(x^2-1) - 2x(x^4-3)}{(x^2+1)^2} < 0$   
 $\Rightarrow (4x^3(x^2-1) - 2x(x^4-3)) < 0 \Rightarrow 2x(x^2-1)(x^2+3) \leq 0 \Rightarrow x(x^2-1)(x^2+3) \leq 0$