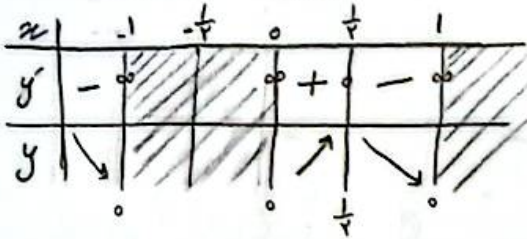


$$f(x) \begin{cases} \sqrt{x-x^2} & x \geq 0 \\ \sqrt{x+x^2} & x < 0 \end{cases} \rightarrow f'(x) \begin{cases} \frac{1-2x}{\sqrt{x-x^2}} & x \geq 0 \\ \frac{1+2x}{\sqrt{x+x^2}} & x < 0 \end{cases}$$

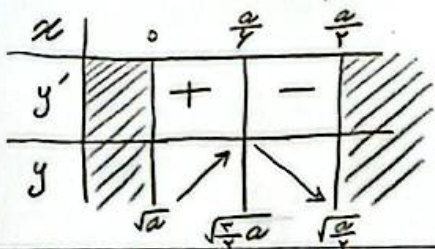


$m=1$
 $n=0$
 $k=k$
 $k+m+n = 2$ ✓

۲

$$f(x) = \sqrt{x} + \sqrt{a-2x} \rightarrow f'(x) = \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{a-2x}} = \frac{\sqrt{a-2x} - \sqrt{x}}{\sqrt{x}\sqrt{a-2x}}$$

$D_f = [0, \frac{a}{2}]$



max $\rightarrow (\frac{a}{4}, \sqrt{\frac{3}{4}a})$

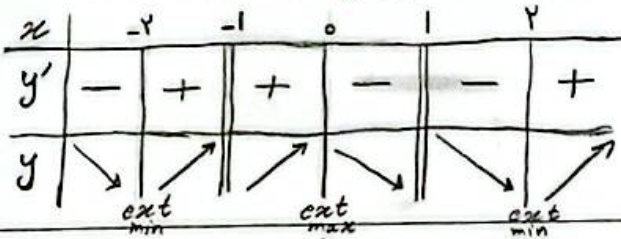
min $\rightarrow (\frac{a}{2}, \sqrt{\frac{a}{2}})$

$\sqrt{\frac{3a}{4}} \times \sqrt{\frac{a}{2}} = \sqrt{\frac{3a^2}{8}} = \sqrt{12} \rightarrow a = 12$

$[a] = [12] = 12$ ✓

۲

$$f(x) \begin{cases} x \in (-\infty, -2) \cup (2, +\infty) & \frac{x^k - 2x^k}{x^k - 1} \\ x \in [-2, 2] & \frac{2x^k - 2x^k}{x^k - 1} \end{cases} \rightarrow f'(x) \begin{cases} \frac{2x^k - 2x^k + 2x^k}{(x^k - 1)^2} \\ \frac{-2x^k + 2x^k - 2x^k}{(x^k - 1)^2} \end{cases}$$



۳ اکسیرم ✓

۲

$x=0 \rightarrow y=0 \rightarrow d=0$

$x=1 \rightarrow y = a + b + c = 1 \xrightarrow{c=0} a + b = 1 \rightarrow 3a + 3b = 3$

$y' = 3ax^2 + 2bx + c$

$x=0 \rightarrow y' = 0 \rightarrow c = 0$

$x=1 \rightarrow y' = 3a + 2b = 0$

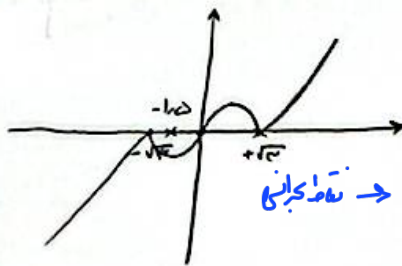
$b = 3 \quad a = -2$

$ab = -6$ ✓

۲

$f(x) = x |\sqrt{3} + x| |\sqrt{3} - x|$

$-\sqrt{3}x - 1,5$



min $\rightarrow x = -1,5 \rightarrow f(-1,5) = -\frac{3}{4} |\sqrt{3} - \frac{3}{4}| |\sqrt{3} + \frac{3}{4}|$

$f(-1,5) = -\frac{9}{8}$

$x = -1,5 \rightarrow f(-1,5) = -1,125$

$x = -1 \rightarrow f(-1) = -2$

$x = 1 \rightarrow f(1) = 2$

$x = \sqrt{3} \rightarrow f(\sqrt{3}) = 0$

$\rightarrow \min = -2$

۱,۵

۵

$$x = -1 \rightarrow y = -x^2 + 3ax + b \rightarrow y = 1 - 3a + b = 1 \rightarrow b - 3a = 0 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} b = 3$$

$$y' = -2x^2 + 3a \rightarrow y' = 3a - 3 = 0 \rightarrow a = 1$$

$$\frac{b}{a} = \frac{3}{1} = 3 \quad f(-1) = 1 \rightarrow 1 + 3a + b = 1 \rightarrow 3a + b = 0 \quad (I)$$

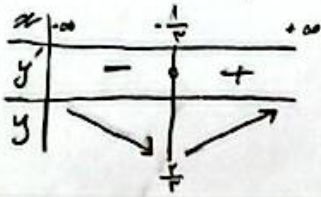
$$\begin{cases} f'(-1) = 0 \\ x = -1 \rightarrow f(x) = -x^2 + 3ax + b \rightarrow f'(-1) = -2x^2 + 3a \end{cases} \quad \frac{b}{a} = \frac{3}{1} = 3$$

$$\rightarrow f'(-1) = -2 - 3a = 0 \rightarrow a = -\frac{1}{3} \xrightarrow{(I)} b = \frac{3}{3}$$

$$\text{میانگین} = \frac{a}{a+1} \quad \frac{a}{a+1} = \frac{1}{3} \rightarrow a = 2 \rightarrow y = \frac{2x+3}{3x+1}$$

$$\text{میانگین قائم} = \frac{1-a}{a+1} \quad \frac{1-a}{a+1} = -\frac{1}{3} \rightarrow a = 2$$

$$y = \frac{2}{3}x^2 + x + \frac{3}{3} \rightarrow y' = 3x + 1 \quad y = 0 \rightarrow \frac{2x+3}{3x+1} = 0 \rightarrow x = -\frac{3}{2}$$

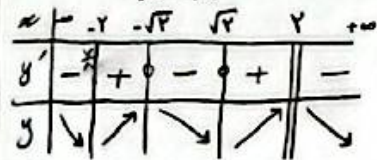


$$\text{میانگین} = \frac{b}{a} = 3 \rightarrow b = 12$$

$$\text{میانگین قائم} \quad 3x^2 + ax + 1 = 0 \quad x = -\frac{1}{3} \rightarrow 1 - \frac{a}{3} + 1 = 2 - \frac{a}{3} = 0 \rightarrow a = 6$$

$$\frac{b}{a} = \frac{12}{6} = 2 \quad \checkmark$$

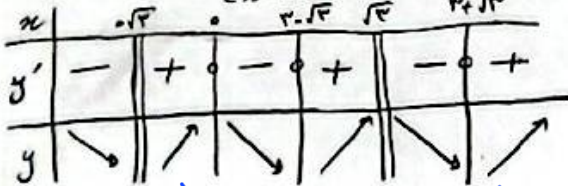
$$f(x) = \frac{x^2}{x^2-1} \rightarrow f'(x) = \frac{2x^2(x^2-1) - 2x^2}{(x^2-1)^2} = \frac{x^4-1}{(x^2-1)^2}$$



$$(-\infty, -1] \quad [-1, 1] \rightarrow \sqrt{2} - (-\sqrt{2}) = 2\sqrt{2} \quad (2, +\infty)$$

(جواب سوال 9)

$$f(x) = \frac{4x^2(x^2-3) - 2x(x^2-3)^2}{(x^2-3)^2} = \frac{4x^4 - 12x^2 + 6x}{(x^2-3)^2}$$

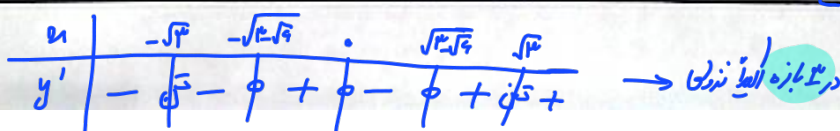


$$(-\infty, -\sqrt{3}) \cup [0, 3-\sqrt{3}] \cup (3+\sqrt{3}, +\infty) \quad \text{از سوال 10}$$

بازه 3

$$4x^4 - 12x^2 + 6x = 0 \rightarrow 2x(2x^3 - 6x + 3) = 0 \rightarrow \{x = 0\}$$

$$\rightarrow 2x^3 - 6x + 3 = 0 \quad x = z \rightarrow z^3 - 3z + \frac{3}{2} = 0 \rightarrow z = \frac{3 \pm \sqrt{3}}{2} \rightarrow \begin{cases} z = \pm \sqrt{3-\sqrt{3}} \\ z = \pm \sqrt{3+\sqrt{3}} \end{cases}$$



$$f(x) = \frac{x^4}{x^3 - 1} \rightarrow f'(x) = \frac{4x^3(x^3 - 1) - 3x^2(x^4)}{(x^3 - 1)^2} = \frac{x^4 - 3x^2}{(x^3 - 1)^2} < 0$$

$$x \neq 1 \rightarrow x^4 - 3x^2 < 0 \rightarrow x^2(x^2 - 3) < 0 \rightarrow 0 < x < \sqrt{3}, x \neq 1$$

$$(0, 1) \rightarrow \text{طول بازه} = 1$$

$$(1, \sqrt{3}) \rightarrow \text{طول بازه} = 2(\sqrt{3} - 1) < 2 \rightarrow \text{طول بازه}^{\min} = 2(\sqrt{3} - 1)$$

تابع در دو بازه $(0, 1)$ و $(1, \sqrt{3})$ ↖ ابتدا نزولی