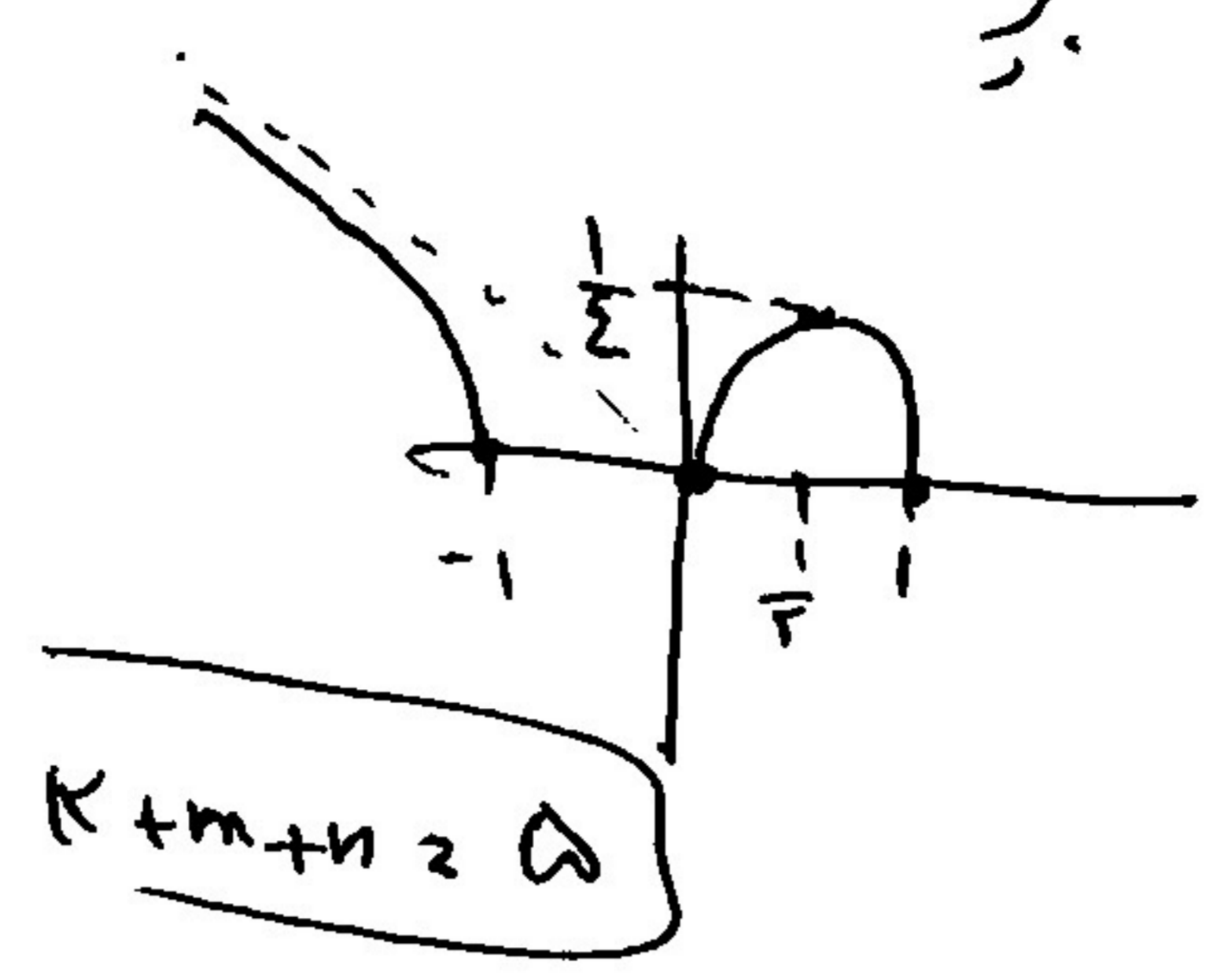


مسئله ۱: دو تابع زیر

$$f(x) = \begin{cases} \sqrt{x-x^2} & x \geq 0 \\ \sqrt{x^2+x} & x < 0 \end{cases}$$

$m=1, n=0, k=2$



D: $x \geq 0, a-x \geq 0 \rightarrow x \in [0, a]$

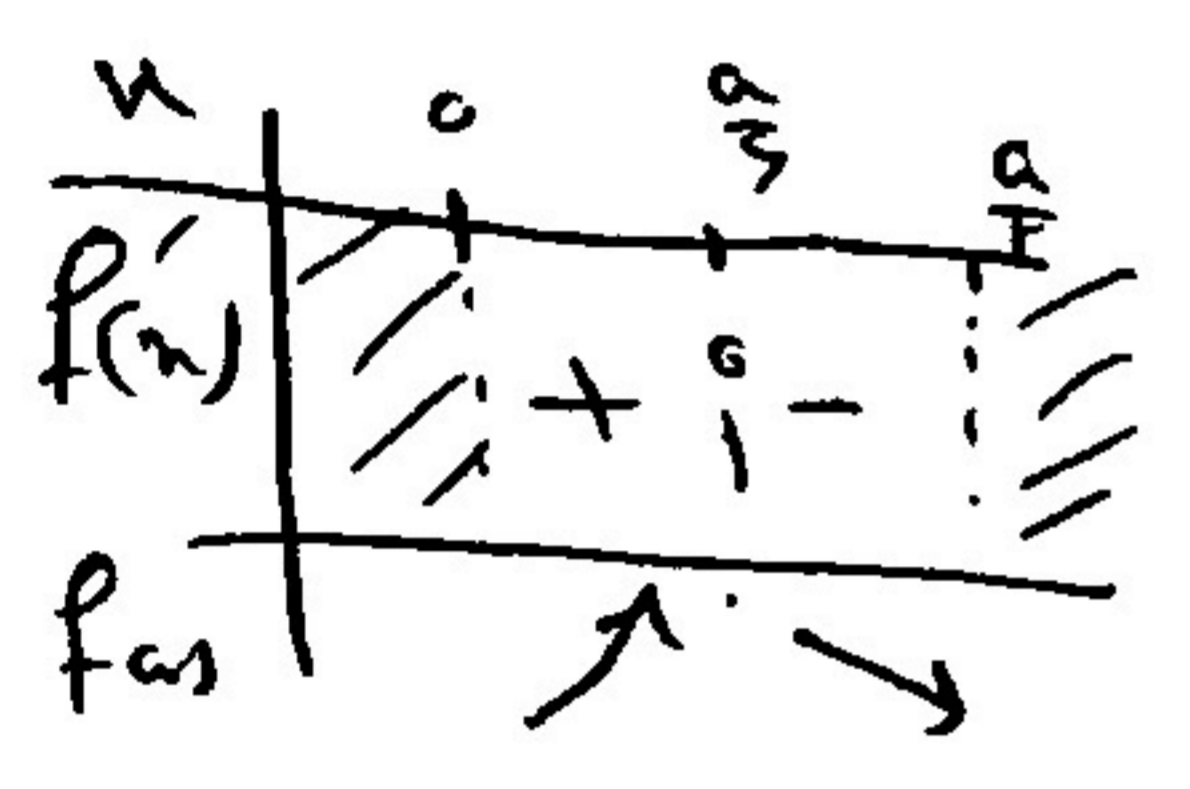
$$f'(x) = \frac{1}{2\sqrt{x}} - \frac{x}{\sqrt{x-x^2}} = 0 \rightarrow x = \frac{a}{2}$$

نقاط بحرانی

$$\begin{cases} x = \frac{a}{2} \rightarrow f(x) = \sqrt{\frac{a}{2}} + \sqrt{\frac{ka}{2}} \text{ max} \\ x = 0 \rightarrow f(x) = \sqrt{a} \\ x = \frac{a}{2} \rightarrow f(x) = \sqrt{\frac{a}{2}} \text{ min} \end{cases}$$

$$\sqrt{\frac{a}{2}} \left(\sqrt{\frac{a}{2}} + \sqrt{\frac{ka}{2}} \right) = \sqrt{ka}$$

$$\frac{ka}{\sqrt{ka}} = \sqrt{ka} \rightarrow a = \frac{ka}{ka} \rightarrow [a] = \frac{ka}{ka}$$



$$f(x) = \frac{x^2}{x-1} \rightarrow f'(x) = \frac{2x(x-1) - x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2}$$

$$\frac{x^2-2x}{(x-1)^2} = 0 \rightarrow x(x-2) = 0 \rightarrow x=0, x=2$$

$$f''(x) = \frac{2x(x-1)^2 - 2(x^2-2x)(x-1)}{(x-1)^4} = \frac{2x(x-1) - 2(x^2-2x)}{(x-1)^3} = \frac{2x^2-2x-2x^2+4x}{(x-1)^3} = \frac{2x}{(x-1)^3}$$

در $x=0$ مشتق دوم $f''(0) = 0$ است.
در $x=2$ مشتق دوم $f''(2) = \frac{4}{1} = 4 > 0$ است.
پس در $x=2$ یک نقطه بحرانی محلی داریم.

۳. اگر فرض کنیم $x > 1$ و $x < 2$ و $x > 2$ در

$$f(0) = 0 \rightarrow a = 0$$

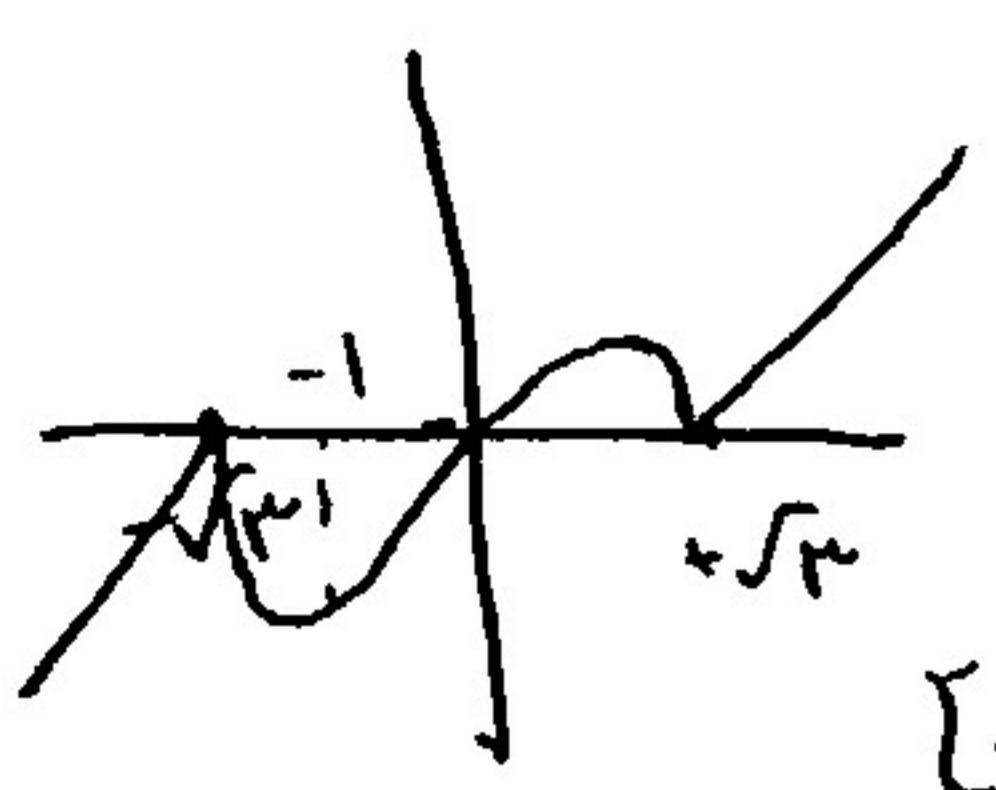
$$f(1) = 1 \rightarrow a + b + c = 1$$

$$f(2) = 2 \rightarrow 4a + 2b + c = 2$$

$$\begin{cases} 2a + b = 1 \\ 2b - 2a = 1 \end{cases} \Rightarrow \begin{cases} a = -1 \\ b = 3 \end{cases}$$

$$f(x) = 2ax^2 + \frac{b}{x} + c = 2a(x)(x-1) = 2ax^2 - 2ax$$

$$ab = -9$$



$$f(x) = 2x\sqrt{x-1} - 2\sqrt{x-1}$$

در بازه $[-1, 1]$ مشتق دوم $f''(x) < 0$ است.
پس در $x=1$ یک نقطه بحرانی محلی داریم.

$$f(x) = 2x - x^2 \rightarrow f'(x) = 2 - 2x \rightarrow x = 1$$

$$f(1) = -1$$

در بازه $[-1, 1]$ مشتق دوم $f''(x) < 0$ است.
پس در $x=1$ یک نقطه بحرانی محلی داریم.

$f(-1) = 1 + \mu a + b = 0 \xrightarrow{a = \frac{1}{\mu}} \boxed{b = 2 + \frac{\mu}{\mu}}$

$f'(x) = -\mu x^{\mu-1} + \mu a x^{\mu-1} \xrightarrow{x = -1} -\mu - \mu a = 0 \rightarrow \boxed{a = -\frac{1}{\mu}}$

$\therefore a = \frac{1}{\mu}, b = 2 + \frac{\mu}{\mu} = \boxed{3}$

$y = \frac{1}{\mu} x^{\mu} + a x + b \rightarrow x = \frac{1}{\mu} \Rightarrow y = \frac{1}{\mu^2} \Rightarrow x = \frac{1}{\mu}$

$f(x) = \frac{1}{\mu} x^{\mu} + a x + b = 0 \rightarrow a = -\frac{1}{\mu}$

$f(x) = \frac{1}{\mu} x^{\mu} + a x + b = 0 \rightarrow \boxed{x = -\frac{\mu}{\mu+1}}$

جانب راست: $x = \frac{1}{\mu} \rightarrow 1 + \frac{1}{\mu} a + 1 = 0 \rightarrow a = -2$

جانب چپ: $y = \frac{b}{\mu} = 3 \rightarrow b = 3 \mu$

$\frac{b}{a} = \frac{3\mu}{-2} = \boxed{-\frac{3\mu}{2}}$

$f'(x) = \frac{\sum x^{\mu} (\mu - 1) - \mu x^{\mu} (\mu \epsilon)}{(\mu - 1)^2} = \frac{\mu x^{\mu} (\mu - 1) - \mu^2 x^{\mu}}{(\mu - 1)^2} = \frac{\mu x^{\mu} (\mu - 1 - \mu)}{(\mu - 1)^2} = \frac{-\mu x^{\mu}}{(\mu - 1)^2}$

x	0	2	$\sqrt[3]{32}$
$f'(x)$	$+$	$+$	$+$
$f(x)$	$+$	$-$	$-$

نکات: $[0, 2]$ نزولی، $[2, \sqrt[3]{32}]$ نزولی، طول تناهی = $\sqrt[3]{32} - 2$

$f(x) = \frac{\sum x^{\mu} (\mu - \mu) - (\mu) (x^{\mu} - \mu)}{(\mu - \mu)^2} = \frac{\mu x^{\mu} (\mu - \mu) - \mu (x^{\mu} - \mu)}{(\mu - \mu)^2} = \frac{\mu x^{\mu} (\mu - \mu) - \mu x^{\mu} + \mu^2}{(\mu - \mu)^2} = \frac{-\mu x^{\mu} + \mu^2}{(\mu - \mu)^2}$

$\frac{1}{\mu} \pm \sqrt{\frac{1}{\mu}} = x^{\mu} \rightarrow x = \sqrt[\mu]{\frac{1}{\mu} \pm \sqrt{\frac{1}{\mu}}}$

(۳ بازه نزولی است)

x	$-\sqrt[3]{\frac{32}{3}}$	0	$\sqrt[3]{\frac{32}{3}}$	$\sqrt[3]{\frac{32}{3}}$	$\sqrt[3]{\frac{32}{3}}$	$\sqrt[3]{\frac{32}{3}}$
$f'(x)$	$+$	$+$	$-$	$+$	$+$	$-$
$f(x)$	$+$	$+$	$-$	$-$	$-$	$-$