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$f(x) = \cos^3(2x) + ax^2 + b$

$\lim_{x\to 0^+} \frac{f(x)}{x} = 0 \rightarrow \lim_{x\to 0^+} \frac{\cos^3(2x)+ax^2+b}{x} = 0 \rightarrow \lim_{x\to 0^+} \frac{1+b}{x} = 0 \rightarrow \boxed{b=-1}$

$\lim_{x\to 0^-} \frac{f'(x)}{x} = 4 \rightarrow \lim_{x\to 0^-} \frac{-6\sin(2x)\cos^2(2x)+2ax}{x} = 4 \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0^-} \frac{-6\times 2x + 2ax}{x} = 4$ ساهان نجفی

$\lim_{x\to 0^+} \frac{f(x)}{x} = 0 \longrightarrow f'_+(0) = 0 \; , \; f(0) = 0 \longrightarrow (b=0)$ ۰٫۵ ①

$\rightarrow \lim_{x\to 0^-} \frac{(2a-12)x}{x} = 2 \rightarrow 2a-12 = 2 \rightarrow \boxed{a=7}$

$\lim_{x\to 0^-} \frac{f'(x)}{x} = 2 \longrightarrow f''(0) = 2 \longrightarrow f'(x) = -6\sin 2x \cos^2 2x + 2ax$ $a+b=6$

$\longrightarrow f''(x) = -6(2\cos 2x\cos^2 2x + \sin 2x \times 2 \times (-\sin 2x)\cos 2x \times 2) + 2a \xrightarrow{x=0} -12+2a=0 \rightarrow a=6$

$\longrightarrow a+b = 6+0 = \boxed{6}$

$d: y = m(mx-1) \longrightarrow x^2-1 = m \longrightarrow x^2 = m+1 \longrightarrow x = \pm\sqrt{m+1}$ ②

$\longrightarrow y' = 2x \longrightarrow \begin{cases} 2\sqrt{m+1} \\ -2\sqrt{m+1} \end{cases} \Longrightarrow -4(m+1) = -1 \longrightarrow m+1 = \frac{1}{4} \longrightarrow m = -\frac{3}{4} \xrightarrow{\text{مجموع عرض نقاط}} = 2m = \boxed{-\frac{3}{2}}$

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$m = \frac{18}{3} = 6 \rightarrow d: 6x - 9 = y$

$m = \frac{6-(-12)}{2.5-(-0.5)} = \frac{18}{3} = 6$, $6 = 6(2.5) + b \rightarrow b = -9$ $\Big\} \rightarrow y = 6x - 9$ ③ ۰٫۵

$\frac{a}{2x-1} = 6x - 9 \longrightarrow a = 12x^2 - 18x - 6x + 9 \Rightarrow 12x^2 - 24x + 9 - a = 0$

$\rightarrow \Delta = 0 \longrightarrow 24\times 24 - 4\times 12(9-a) = 0 \longrightarrow 12 - 9 + a = 0 \longrightarrow a = 7$ $f(5) = \frac{-1}{3}$

$\rightarrow f(x) = \frac{7}{2x-1} \xrightarrow{x=5} \frac{7}{9} = f(5)$

خط بر منحنی مماس است $\rightarrow \Delta = 0$

$\frac{a}{2x-1} = (6x-9) \rightarrow 12x^2 - 18x - 6x + 9 - a = 0 \rightarrow 12x^2 - 24x + 9 - a = 0$

$\Delta = 0 \rightarrow (24)^2 - 4(12)(9-a) = 0 \rightarrow 576 - 432 + 48a = 0 \rightarrow 48a = -144 \rightarrow a = -3$

$x=1 \longrightarrow 2+b = \frac{1+a}{a+1} = 1 \longrightarrow \boxed{b=-1}$ ④

$y'_1 = 2 \; , \; y'_2 = \frac{1-a^2}{(ax+1)^2} \xrightarrow{x=1} \frac{1-a^2}{(a+1)^2} = \frac{(1-a)(1+a)}{(1+a)(1+a)} = \frac{1-a}{1+a} = 2$

$\longrightarrow 1-a = 2+2a \longrightarrow 3a = -1 \longrightarrow a = \frac{1}{3} \longrightarrow a = \frac{-1}{3} \longrightarrow \frac{-1}{3} + 1 = \boxed{\frac{2}{3}}$

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ماهان نجفی

۵) $\sin x + \frac{1}{2}\cos 2x = \frac{3}{2}\sin x \longrightarrow \frac{1}{2}\cos x = \frac{1}{2}\sin x \xrightarrow{0 \leq x \leq \pi} x = \frac{\pi}{4}$

$f'(x) = \cos x - \frac{1}{2}\sin x \xrightarrow{x=\frac{\pi}{4}} \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4} =$ شیب خط

$f(\frac{\pi}{4}) = g(\frac{\pi}{4}) = \frac{3}{2} \times \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{4} \longrightarrow$ معادله: $\frac{\sqrt{2}}{4}(x - \frac{\pi}{4}) = y - \frac{3\sqrt{2}}{4}$

$\xrightarrow{y=0} \frac{\sqrt{2}}{4}(x - \frac{\pi}{4}) = \frac{-3\sqrt{2}}{4} \longrightarrow \boxed{x = \frac{\pi}{4} - 3}$

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۶) $f'(x) = 6x^2 - 6x - 12 = 0 \longrightarrow$ $\begin{array}{ccccc} & -1 & & 2 & \\ + & & - & & + \end{array}$

$f(-1) = -2 - 3 + 12 + 1 = 8 = f(-1)$

$f(2) = 16 - 12 - 24 + 1 = -19 = f(2)$

$\longrightarrow m_{AB} = \frac{8+19}{-1-2} = \frac{27}{-3} = -9 \longrightarrow 6x^2 - 6x - 12 = -9 \longrightarrow 6x^2 - 6x - 3 = 0 \ (\Delta > 0)$

دو نقطه روی منحنی دارای شیب مساوی با خط AB هستند و موازی آن هستند

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۷) $y' = 3kx^2 + (2k+2)x \longrightarrow y'' = 6kx + 2k + 2$

$\longrightarrow 6kx = -2k - 2 \longrightarrow x = \frac{-2k-2}{6k} = \frac{-k-1}{3k} \longrightarrow \frac{-k-1}{3k} < 0 \longrightarrow$ $\begin{array}{ccccc} & -1 & & 0 & \\ - & & + & & - \end{array}$

$\longrightarrow y = x^2(kx + k + 1) \longrightarrow (\frac{k+1}{3k})^2(\frac{-k-1}{3} + k + 1) = (\frac{k+1}{3k})^2(\frac{2k+2}{3}) > 0 \longrightarrow$ $\begin{array}{ccc} & -1 & \\ - & & + \end{array}$

اشتراک $k > 0 \longrightarrow$ به ازای همه مقادیر مثبت

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۸) در نقطه عطف خط مماس از نمودار می‌گذرد

$x = -1 \longrightarrow -1 + a - b - 1 = -4 \longrightarrow \boxed{a - b = -2}$

$y' = 3x^2 + 2ax + b \longrightarrow y'' = 6x + 2a \xrightarrow{x=-1} -6 + 2a = 0 \longrightarrow a = 3$

$\longrightarrow 3 - b = -2 \longrightarrow b = 5 \longrightarrow \boxed{\frac{a}{b} = \frac{3}{5}}$

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ماهان کیانی

(۹)

$f(0)=c=4 \longrightarrow f(x)=x^3+ax^2+bx+4 \longrightarrow f'(x)=3x^2+2ax+b$

$\xrightarrow{x=0} f'(0)=b=0 \longrightarrow f(x)=x^3+ax^2+4 \longrightarrow f'(x)=3x^2+2ax=0$

$=x(3x+2a)=0 \longrightarrow 3x=-2a \longrightarrow x=\frac{-2a}{3} \longrightarrow f(-\frac{2a}{3})=\frac{-8a^3}{27}+\frac{4a^3}{9}+4=0$

$\frac{-8a^3}{27}+\frac{12a^3}{27}+4=0 \longrightarrow \frac{4a^3}{27}=-4 \longrightarrow a^3=-27 \longrightarrow a=-3$

$\longrightarrow -\frac{2}{3}(-3)=\boxed{2=x}$

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(۱۰)

$f'(x)=4x^3-12x=4x(x^2-3) \longrightarrow$

$-\sqrt{3}$		0		$\sqrt{3}$	
$-$	$+$		$-$		$+$
	min			min	

$f(\sqrt{3})=9-6\times3+5=-4=f(\sqrt{3})$

$f(-\sqrt{3})=9-6\times3+5=-4=f(-\sqrt{3}) \longrightarrow A(\sqrt{3},-4), B(-\sqrt{3},-4)$

$f''(x)=12x^2-12=0 \longrightarrow \begin{cases} x=1 \longrightarrow f(1)=0 \longrightarrow D(1,0) \\ x=-1 \longrightarrow f(-1)=0 \longrightarrow C(-1,0) \end{cases}$

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زاویه صفر درجه

این دو پاره خط AB و CD موازی هم هستند بین آنها هیچ زاویه ای ایجاد نمی شود!