

$$\lim_{x \rightarrow 0^+} \frac{f(x)}{x} = 0 \rightarrow f'(0) = 0, f(0) = 0 \rightarrow \boxed{b=0} \quad (1)$$

$$\lim_{x \rightarrow 0} \frac{f'(x)}{x} = 2 \rightarrow f''(0) = 2 \rightarrow f'(x) = -4 \sin x \cos^2 x + 4ax$$

$$\rightarrow f''(x) = -4(\cos^2 x \cos^2 x + \sin^2 x \times 2x(-\sin x) \cos^2 x) + 4a \xrightarrow{x=0} -4 + 4a = 0 \rightarrow a = 1$$

$$\rightarrow a + b = 1 + 0 = 1$$

$$d = y = m(m^2 - 1) \rightarrow x^2 - 1 = m \rightarrow x^2 = m + 1 \rightarrow x = \pm \sqrt{m+1} \quad (2)$$

$$\rightarrow y' = 2x \rightarrow \frac{2\sqrt{m+1}}{-2\sqrt{m+1}} \Rightarrow -f'(m+1) = 1 \rightarrow m+1 = \frac{1}{f'} \rightarrow m = \frac{-2}{f'} \rightarrow \text{مجموع درجه} = 2m = \frac{-4}{f'} \rightarrow \boxed{\frac{-4}{f'}}$$

$$m = \frac{1}{f'} = 2 \rightarrow d: 4x - 9 = y \quad (3)$$

$$\frac{a}{f_{n-1}} = 4x - 9 \rightarrow a = 12x^2 - 12x - 4x + 9 \rightarrow 12x^2 - 16x + 9 = a = 0$$

$$\rightarrow \Delta = 0 \rightarrow 16x^2 - 16x + 9 = 0 \rightarrow 16 - 9 + a = 0 \rightarrow a = 7$$

$$\rightarrow f(x) = \frac{7}{x-1} \xrightarrow{x=0} \frac{7}{-1} = f(0)$$

$$x=1 \rightarrow 7 + b = \frac{7+a}{a+1} = 1 \rightarrow \boxed{b = -1} \quad (4)$$

$$y' = 2, y'' = \frac{1-a^2}{(a+1)^2} \xrightarrow{x=1} \frac{1-a^2}{(a+1)^2} = \frac{(1-a)(1+a)}{(1+a)(1+a)} = \frac{1-a}{1+a} = 2$$

$$\rightarrow 1-a = 2+2a \rightarrow 3a = -1 \rightarrow a = \frac{-1}{3} \rightarrow \frac{-1}{3} + 1 = \boxed{\frac{2}{3}}$$

$$\sin x + \frac{1}{4} \cos x = \frac{3}{4} \sin x \rightarrow \frac{1}{4} \cos x = \frac{1}{4} \sin x \xrightarrow{0 < x < \pi} x = \frac{\pi}{4} \quad (5)$$

$$f'(x) = \cos x - \frac{1}{4} \sin x \xrightarrow{x = \frac{\pi}{4}} \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4} \text{ مثبت}$$

$$f\left(\frac{\pi}{4}\right) = g\left(\frac{\pi}{4}\right) = \frac{3}{4} \cdot \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{8} \rightarrow \text{اذا: } \frac{\sqrt{2}}{4} \left(x - \frac{\pi}{4}\right) = y - \frac{3\sqrt{2}}{8}$$

$$y = 0 \rightarrow \frac{\sqrt{2}}{4} \left(x - \frac{\pi}{4}\right) = -\frac{3\sqrt{2}}{8} \rightarrow x = \frac{\pi}{4} - \frac{3}{2}$$

$$f(x) = 4x^2 - 4x - 12 = 0 \rightarrow \begin{matrix} - & + & - \\ \downarrow & \downarrow & \downarrow \\ 4 & -4 & -12 \end{matrix} \rightarrow f(-1) = -2 - 4 + 12 + 1 = 7 = f(-1) \quad (6)$$

$$f(2) = 16 - 8 - 12 + 1 = -13 = f(2)$$

$$\rightarrow m_{AB} = \frac{1+11}{-1-2} = \frac{12}{-3} = -4 \rightarrow 4x^2 - 4x - 12 = -4 \rightarrow 4x^2 - 4x - 8 = 0 \quad (\Delta < 0)$$

نقطه‌های منتهی در این فاصله، در خط AB هستند و موازی آن هستند

$$y' = 3kx^2 + (2k+2)x \rightarrow y'' = 6kx + 2k + 2 \quad (7)$$

$$\rightarrow 6kx = -2k - 2 \rightarrow x = \frac{-2k-2}{6k} = \frac{-k-1}{3k} \rightarrow \frac{-k-1}{3k} < 0 \rightarrow \frac{-}{+} = \frac{-}{+}$$

$$\rightarrow y = x^2(kx + k + 1) \rightarrow \left(\frac{k+1}{3k}\right)^2 \left(\frac{-k-1}{3} + k + 1\right) = \left(\frac{k+1}{3k}\right)^2 \left(\frac{2k+2}{3}\right) > 0 \rightarrow \frac{+}{-}$$

نتیجه $k > 0$ به ازای هیچ مقدار صحیح مثبتی

$$x = -1 \rightarrow -1 + a - b - 1 = -2 \rightarrow a - b = -2 \quad (8) \text{ در نقطه منتهی در فاصله موازی آن نزدیک}$$

$$y' = 3x^2 + 2ax + b \rightarrow y'' = 6x + 2a \xrightarrow{x = -1} -4 + 2a = 0 \rightarrow a = 2$$

$$\rightarrow 2 - b = -2 \rightarrow b = 4 \rightarrow \frac{a}{b} = \frac{2}{4}$$

$$f(0) = c = 4 \rightarrow f(x) = x^3 + ax^2 + bx + 4 \rightarrow f'(x) = 3x^2 + 2ax + b \quad (9)$$

$$x=0 \rightarrow f'(0) = b = 0 \rightarrow f(x) = x^3 + ax^2 + 4 \rightarrow f'(x) = 3x^2 + 2ax = 0$$

$$= x(3x + 2a) = 0 \rightarrow 3x = -2a \rightarrow x = \frac{-2a}{3} \rightarrow f\left(\frac{-2a}{3}\right) = \frac{-1a^3}{27} + \frac{4a^3}{9} + 4 = 0$$

$$\frac{-1a^3}{27} + \frac{4a^3}{9} + 4 = 0 \rightarrow \frac{3a^3}{27} = -4 \rightarrow a^3 = -12 \rightarrow a = -\sqrt[3]{12}$$

$$\rightarrow \frac{-2}{3}(-\sqrt[3]{12}) = \sqrt[3]{2} = x$$

$$f'(x) = 3x^2 - 12x = 3x(x - 4) \rightarrow \begin{matrix} -\sqrt{3} & 0 & \sqrt{3} \\ -1 & + & 1 \\ \downarrow & & \downarrow \\ \text{min} & & \text{min} \end{matrix} \quad (10)$$

$$f(\sqrt{3}) = 9 - 12\sqrt{3} + 4 = -12\sqrt{3} = f(\sqrt{3})$$

$$f(-\sqrt{3}) = 9 - 12(-\sqrt{3}) + 4 = 12\sqrt{3} + 13 = f(-\sqrt{3}) \rightarrow A(\sqrt{3}, -12\sqrt{3}), B(-\sqrt{3}, 12\sqrt{3} + 13)$$

$$f''(x) = 6x - 12 = 0 \rightarrow x = 2 \rightarrow f(2) = 8 - 24 + 4 = -12 \rightarrow D(2, -12)$$

$$x = -2 \rightarrow f(-2) = -8 - 24 + 4 = -28 \rightarrow E(-2, -28)$$

اسی دو بارہ خانہ AB, CD, E میں آئے ہیں۔