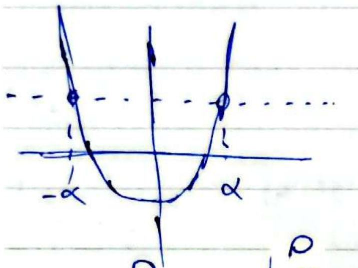
D۲۷   دوازدهم A   ۱۸٫۵   پارسا [illegible]

۱- $f'(x)=3\times 2\times \cos^2(2x)\times(-\sin 2x)$ و $\lim \frac{f(x)-f(0)}{x-0}=0 \Rightarrow b=0$

$+2ax \Rightarrow$ if $x=0 \Rightarrow f'(0)=0$ $\lim \frac{f'(x)-f'(0)}{x-0}=2 \Rightarrow f''(0)=2$ -0.5

$\Rightarrow f''(0)=2a \Rightarrow a=1 \Rightarrow a+b=1$ (جواب صفر بود)



$f'(\alpha)\times f'(-\alpha)=-1$ ، $f(x)=x^2+1 \Rightarrow f'(x)=2x-2$

$\Rightarrow -4\alpha^2=-1 \Rightarrow \alpha^2=\frac{1}{4} \Rightarrow \alpha=\pm\frac{1}{2}$ ۲

$\Rightarrow f(x) \Rightarrow$ if $x=\pm\frac{1}{2} = \frac{1}{4}-1=\frac{-3}{4}\times 2=-\frac{3}{2}$

۳- $m=\frac{6-(-12)}{2.5-(-0.5)}=6 \Rightarrow y-6=6(x-2.5)$ ⟸ شیب = امتداد مماس

$y=6x-9 \Rightarrow 6x-9=\frac{a}{2x-1} \longrightarrow 12x^2-24x+9-a=0 \Rightarrow \Delta=0$ ۲

$\Rightarrow (-24)^2-4(12)(9-a)=0 \Rightarrow 48a=-144 \Rightarrow a=-3 \Rightarrow f(5)$

$=\frac{-3}{9}=-\frac{1}{3}$

۴- if $x=1 \Rightarrow 2x+b=\frac{x+a}{ax+1}$ ، $\frac{1\times(ax+1)-a(x+a)}{(ax+1)^2}=2 \Rightarrow \frac{1-a^2}{(a+1)^2}=2$

$\Rightarrow 2a^2+4a+2-1+a^2=0 \Rightarrow (a+1)(3a+1)=0$ $a=-1$ یا $a=-\frac{1}{3}$

$\Rightarrow y=\frac{x-\frac{1}{3}}{-\frac{1}{3}x+1}=\frac{3x-1}{-3+x}$ قابل قبول نیست ⟸ $\frac{0}{0}$ $\xrightarrow{\text{if } x=1} y=1$ ۲

$\Rightarrow 2+b=1 \Rightarrow b=-1$ ⟸ $a-b=-\frac{1}{3}+1=\frac{2}{3}$

فقط ۱/۳ [illegible]

$f(x) = g(x) \Rightarrow \sin x + \frac{1}{2}\cos x = \frac{3}{2}\sin x \Rightarrow \frac{1}{2}\sin x = \frac{1}{2}\cos x \Rightarrow \frac{\sin x}{\cos x} = 1$ -5

$\Rightarrow x = \frac{\pi}{4} \Rightarrow \left|\begin{matrix}\frac{\pi}{4} \\ \frac{3\sqrt{2}}{4}\end{matrix}\right.$, $f'(x) = \cos x - \frac{1}{2}\sin x \Rightarrow$ if $x = \frac{\pi}{4}$

$f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4} \Rightarrow d: y - \frac{3\sqrt{2}}{4} = \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)$

$\Rightarrow y = 0 \Rightarrow -\frac{3\sqrt{2}}{4} = \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right) \Rightarrow x = \frac{\pi}{4} - 3$

$f'(x) = 6x^2 - 6x - 12 = 0 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x = -1,\ x = 2$ و $m_{AB} = \frac{-19-8}{2-(-1)} = -9$ -6

$\Rightarrow 6x^2 - 6x - 12 = -9 \Rightarrow 6x^2 - 6x - 3 = 0 \xrightarrow{\Delta > 0}$ ۲ جواب

$y = kx^3 + (k+1)x^2 \Rightarrow y' = 3kx^2 + 2(k+1)x \Rightarrow y'' = 6kx + 2k + 2 = 0$ -7

$k\left(\frac{-(k+1)}{3k}\right)^3 + (k+1)\left(\frac{-(k+1)}{3k}\right)^2 > 0 \Rightarrow \frac{-(k+1)^3 + 3(k+1)^3}{27k^2} > 0$

$\Rightarrow \frac{2(k+1)^3}{27k^2} > 0$; $-1,\ 0$: $-\ +\ +$

و $6kx = -2k - 2 \Rightarrow x = \frac{-(k+1)}{3k} < 0 \Rightarrow \frac{k+1}{3k} > 0 \Rightarrow$ $+\ -\ +$

$\Rightarrow k \in (0, +\infty)$ ⇐ جواب مشترک بازه‌ها

$y = x^3 + ax^2 + bx - 1 \Rightarrow y' = 3x^2 + 2ax + b \Rightarrow y'' = 6x + 2a$ -8

$y'' = 0 \Rightarrow 6x + 2a = 0 \Rightarrow 3x + a = 0 \Rightarrow x = -\frac{a}{3}$; $-\frac{a}{3} = -1 \Rightarrow a = 3$

$y = x^3 + 3x^2 + bx - 1 \Rightarrow$ if $x = -1 \Rightarrow -1 + 3 - b - 1 = -4 \Rightarrow b = 5$

$\frac{a}{b} = \frac{3}{5} = 0.6$

D

$f(x) = x^3 + ax^2 + bx + c \to f(0) = 4 \to c = 4 \to f'(x) = 3x^2 + 2ax + b$ _9

$\to f'(0) = 0 \to b = 0 \to f(x) = x^3 + ax^2 + 4 \to f'(x) = 3x^2 + 2ax$

$\to x(3x + 2a) = 0 \;,\; x = 0 \;,\; \left(x = -\frac{2a}{3}\right) \to \min$ نسبی

6

$f\left(-\frac{2a}{3}\right) = 0 \to \left(-\frac{2a}{3}\right)^3 + a\left(-\frac{2a}{3}\right)^2 + 4 = 0 \to a = -3 \to -\frac{2a}{3}$

$= \frac{-2(-3)}{3} = 2$

$f(x) = x^4 - 6x^2 + 8 \to f'(x) = 4x^3 - 12x = 0 \to x^3 - 3x = 0$ _10

$x(x^2 - 3) = 0 \to$

x		$-\sqrt{3}$		0		$\sqrt{3}$	
$f(x)$	$-$		$+$		$-$		$+$
$f'(x)$	$\searrow$	-4	$\nearrow$		$\searrow$	-4	$\nearrow$

$\to f''(x) = 12x^2 - 12 = 0$

$x^2 - 1 = 0 \to x = \pm 1$

$A\begin{vmatrix} -\sqrt{3} \\ -4 \end{vmatrix}$ $\quad B\begin{vmatrix} \sqrt{3} \\ -4 \end{vmatrix}$

$C\begin{vmatrix} 1 \\ 0 \end{vmatrix}$ $\quad D\begin{vmatrix} -1 \\ 0 \end{vmatrix}$

4

$m_{AB} = 0$

$m_{CD} = 0$

* دو نقطه با عرض برابر ← امتیاز دارد

سؤال ۱۱

$f(x) = \cos^3(2x) + ax^2 + b$

$\lim_{x\to 0^+} \frac{f(x)}{x} = 0 \to \lim_{x\to 0^+} \frac{\cos^3(2x) + ax^2 + b}{x} = 0 \to \lim_{x\to 0^+} \frac{1+b}{x} = 0 \to \boxed{b = -1}$

$\lim_{x\to 0^-} \frac{f'(x)}{x} = 2 \to \lim_{x\to 0^-} \frac{-6\sin(2x)\cos^2(2x) + 2ax}{x} = 2 \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0^-} \frac{-6\times 2x + 2ax}{x} = 2$

$\to \lim_{x\to 0^-} \frac{(2a-12)x}{x} = 2 \to 2a - 12 = 2 \to \boxed{a = 7}$ $\quad a + b = 6$