

$$\lim_{x \rightarrow 0^+} \frac{f(x)}{x} = 0 = \frac{0}{0} \rightarrow f(0) = 1 + b = 0 \rightarrow b = -1$$

$$\lim_{x \rightarrow 0^-} \frac{f'(x)}{x} = 0 = \frac{0}{0} \rightarrow f'(x) = 3 \cos^2(2x) \times (-\sin 2x) \times 2 + 2xa$$

$$\rightarrow \lim_{x \rightarrow 0^-} \frac{-12x \cos^2(2x) + 2xa}{x} = -12 \cos^2(2x) + 2a = -12 + 2a = 2 \rightarrow a = 7$$

$$a + b = 7 - 1 = \boxed{6}$$

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$$f(x) = x^2 - 1 \rightarrow f'(x) = 2x$$

$$f'(x_1) = 2x_1$$

$$f'(x_2) = 2x_2$$

$$f'(x_1) \times f'(x_2) = 4x_1x_2 = -1 \rightarrow x_1x_2 = -\frac{1}{4} \xrightarrow{x_1 = -x_2} -x_1^2 = -\frac{1}{4} \rightarrow \begin{cases} x_1 = \frac{1}{2} \\ x_2 = -\frac{1}{2} \end{cases}$$

$$f(x_1) = -\frac{3}{4} \rightarrow f(x_1) + f(x_2) = \boxed{-\frac{3}{2}}$$

$$f(x_2) = -\frac{3}{4}$$

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$$f(x) = a(2x-1)^{-1} \rightarrow f'(x) = -2a(2x-1)^{-2}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{7 - (-12)}{1/2 - (-1/2)} = 4 \rightarrow y = 4x - 9$$

$$\frac{a}{2x-1} = 4x-9 \rightarrow 12x^2 - 2ax + 9 - a = 0 \rightarrow \Delta = 4a^2 - 4(9-a) \rightarrow a = -3$$

$$f(x) = \frac{-3}{2x-1} \rightarrow f(2) = \frac{-3}{4-1} = \boxed{-\frac{1}{3}}$$

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$$y = 2x + b \rightarrow y' = 2$$

$$y = \frac{x+a}{ax+1} \rightarrow y' = \frac{1-a^2}{(ax+1)^2}$$

$$\frac{a+1}{a+1} = 2 + b = 1 \rightarrow b = -1$$

$$\frac{1-a^2}{(ax+1)^2} = 2 \rightarrow \frac{1-a^2}{a^2+2a+1} = 2 \rightarrow 1-a^2 = 2a^2+4a+2 \rightarrow 3a^2+4a+1=0 \rightarrow \begin{cases} a = -1 \text{ GOI} \\ a = -\frac{1}{3} \checkmark \end{cases}$$

$$a - b = -\frac{1}{3} - (-1) = \boxed{\frac{2}{3}}$$

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$$f(x) = g(x) \rightarrow \sin x + \frac{1}{4} \cos x = \frac{\sqrt{2}}{4} \sin x \rightarrow \cos x = \sin x \rightarrow x = \frac{\pi}{4} \rightarrow y = \frac{\sqrt{2}}{4}$$

$$f'(x) = \cos x - \frac{1}{4} \sin x = \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4}$$

$$y = \frac{\sqrt{2}}{4} x + b \rightarrow \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4} \cdot \frac{\pi}{4} + b \rightarrow b = \frac{(4-\pi)\sqrt{2}}{16}$$

$$y = \frac{\sqrt{2}}{4} x + \frac{(4-\pi)\sqrt{2}}{16} \rightarrow \frac{(4-\pi)\sqrt{2}}{16} = \frac{\sqrt{2}}{4} x \rightarrow x = \boxed{\frac{4-\pi}{4}}$$

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$$f(x) = 2x^3 - 3x^2 - 12x + 1 \rightarrow f'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1)$$

$x$	-1	2	
$y'$	+	-	+
$y$	↖	↘	↗
	1	-12	

$$m = \frac{\Delta y}{\Delta x} = \frac{1 - (-12)}{-1 - 2} = -9$$

$$y = -9x - 1$$

$$f'(x) = 6x^2 - 6x - 12 = -9 \rightarrow f'(x) = 6x^2 - 6x - 3$$

$$\Delta > 0 \rightarrow \boxed{\text{قطب 2}}$$

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$$f(x) = kx^3 + (k+1)x^2 \rightarrow f'(x) = 3kx^2 + 2(k+1)x \rightarrow f''(x) = 6kx + 2k + 2$$

$$f''(x) = 6kx + 2k + 2 = 0 \rightarrow x = \frac{-k-1}{3k} < 0 \rightarrow \frac{k+1}{3k} > 0$$

$x$	-1	0	
$y'$	+	-	+
$y$	↖	↘	↗

$$f(x) > 0 \rightarrow kx^3 + (k+1)x^2 > 0 \rightarrow k \frac{(k+1)^3}{27k^3} + (k+1) \frac{(k+1)^2}{9k^2} > 0$$

$$\rightarrow \frac{4(k+1)^3}{27k^2} > 0$$

$x$	-1	0	
$y'$	+	-	+
$y$	↖	↘	↗

موج (مقدار مثبت)

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$$f(x) = x^3 + ax^2 + bx - 1 \rightarrow f(-1) = -1 + a - b - 1 = 4 \rightarrow a - b = 6$$

$$f'(x) = 3x^2 + 2ax + b \rightarrow f'(-1) = 3 - 2a + b = 0 \rightarrow b - 2a = -3$$

$$a = -3$$

$$b = -9$$

$$\frac{a}{b} = \frac{1}{3}$$

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$$f(0) = c = 4$$

$$f'(x) = 3x^2 + 2ax + b \rightarrow f'(0) = b = 0$$

$$\rightarrow \begin{cases} f(x) = x^3 + ax^2 + 4 \\ f'(x) = 3x^2 + 2ax \end{cases}$$

$$f'(x) = 3x^2 + 2ax \rightarrow \begin{cases} x = 0 \\ x = -\frac{2a}{3} \end{cases}$$

$$f(-\frac{2a}{3}) = \frac{-11a^3}{27} + \frac{4a^3}{9} + 4 = \frac{4a^3}{27} + 4 = 0 \rightarrow a^3 = -27 \rightarrow a = -3$$

$$x = -\frac{2a}{3} = 2$$

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$$f(x) = x^3 - 4x^2 + 5 \rightarrow f'(x) = 3x^2 - 8x = 3x(x - \frac{8}{3})$$

$x$	$\frac{A}{\sqrt{3}}$	0	$\frac{B}{\sqrt{3}}$	
$y'$	-	+	-	+
$y$	↘	↗	↘	↗
	-4		-4	

$$f''(x) = 6x - 8 = 12(x - \frac{2}{3})(x - \frac{4}{3})$$

$x$	$\frac{C}{3}$	$\frac{D}{3}$	
$y''$	+	-	+
$y$	↖	↘	↗

$$m_{AB} = 0$$

$$m_{DC} = 0$$

یکدیگر را قطع نمی کنند  
در خط مماسی اند

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