

$$\rightarrow yx - y = x^2 + x + 2 \rightarrow x^2 + (1-y)x + (y+2) = 0 \rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\rightarrow \frac{y-1 \pm \sqrt{y^2 - 4y + 1 - 4y - 8}}{2} \rightarrow y^2 - 8y - 7 \geq 0 \rightarrow \frac{-1}{+} \phi - \frac{7}{+} \phi +$$

$$\rightarrow R = (-\infty, -1] \cup [7, +\infty)$$

الف) $x_s = -\frac{b}{2a} = \frac{5}{2} \rightarrow y_s = \frac{10}{2} - \frac{5^2}{2} + \frac{1}{2} = -\frac{19}{2}$ $\rightarrow x^2 - 3x^2 + 2x + 1 \in R(-\infty, +\infty)$
 $\rightarrow R = [-\frac{19}{2}, +\infty)$ ($a > 0$)
 ب) $x_s = -\frac{b}{2a} = 2 \rightarrow y_s = 4 \rightarrow R = (-\infty, 4]$
 $\rightarrow R = (-\infty, 4]$

الف) $R = R - \{\text{مجاذب افقی}\} = R - \{\frac{a}{c}\}$
 $\Rightarrow R = R - \{\frac{3}{2}\}$
 ب) $\rightarrow R = R - \{4\} \xrightarrow{\text{مخرج}} R = [0, +\infty) - \{2\}$
 $\rightarrow R = [\frac{1}{2}, +\infty)$

روش اول: $\rightarrow yx^2 - y = 3x^2 + 4 \rightarrow (3-y)x^2 + (y+4) = 0 \rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{\pm \sqrt{-4(3-y)(y+4)}}{2(3-y)} \rightarrow y \neq 3$
 $\rightarrow -4(3-y)(y+4) \geq 0 \rightarrow \frac{-4}{+} \frac{3}{-} \frac{b}{+} \rightarrow (-\infty, -4] \cup (3, +\infty)$
 روش دوم: $\rightarrow x=0 \rightarrow y=-4$
 $\rightarrow x=\infty \rightarrow y=3$
 $\rightarrow R = (-\infty, -4] \cup (3, +\infty)$

روش اول: $\rightarrow y \sin x + 3y = 2 \sin x - 1 \rightarrow (2-y) \sin x - (3y+1) = 0$
 $\rightarrow \sin x = \frac{3y+1}{2-y} \Rightarrow -1 \leq \frac{3y+1}{2-y} \leq 1$
 $\rightarrow \frac{3y+1}{2-y} \leq 1 \rightarrow \frac{3y+1}{2-y} - 1 \leq 0 \rightarrow \frac{3y+1 - (2-y)}{2-y} \leq 0 \rightarrow \frac{4y-1}{2-y} \leq 0$
 $\rightarrow \frac{4y-1}{2-y} \geq -1 \rightarrow \frac{4y-1}{2-y} + 1 \geq 0 \rightarrow \frac{4y-1 + 2-y}{2-y} \geq 0 \rightarrow \frac{3y-1}{2-y} \geq 0$
 $\rightarrow \sin x = 1 \rightarrow \frac{1}{2}$
 $\rightarrow \sin x = -1 \rightarrow -\frac{1}{2}$
 $\rightarrow \sin x = 0 \rightarrow -\frac{1}{2}$
 $\rightarrow R = [-\frac{3}{2}, \frac{1}{2}] \leftarrow \cap \leftarrow$

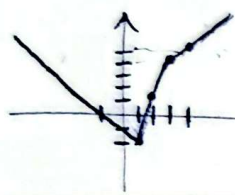
$$f(x) = \frac{x^2 - x + \frac{1}{\varepsilon}}{x^2 - x + 1} = \frac{(x - \frac{1}{\varepsilon})^2}{(x - \frac{1}{\varepsilon})^2 + \frac{\varepsilon}{\varepsilon^2}}$$

$\square = +\infty \rightarrow 1$
 $\square = 0 \rightarrow 0$

$\Delta < 0$
 $a > 0 \rightarrow \oplus \rightarrow D_f = \mathbb{R}$

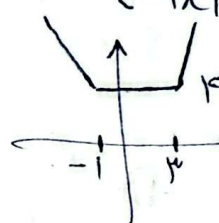
$R = (-\infty, 0] \cup (1, +\infty)$
 $= \mathbb{R} - (0, 1]$

ج) $y = \begin{cases} x+1 & x > 2 \\ 3x-2 & 1 < x \leq 2 \\ -x-1 & x \leq 1 \end{cases}$



$\Rightarrow R = [-1, +\infty)$

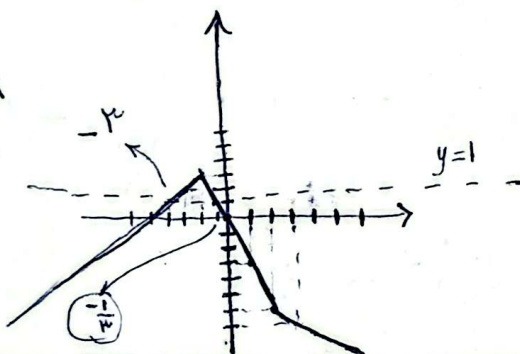
د) $y = \begin{cases} 2x-2 & x > 2 \\ 1 & -1 < x \leq 2 \\ -2x+2 & x \leq -1 \end{cases}$



$\Rightarrow R = [1, \infty)$

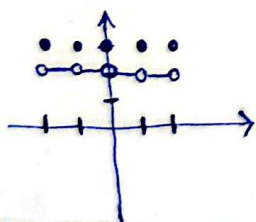
$|x-2| - |2x+2|$

$\begin{cases} -x-2 & x > 2 \\ -3x & -1 < x \leq 2 \\ x+2 & x \leq -1 \end{cases}$



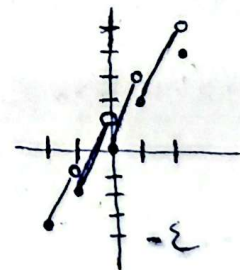
$\Rightarrow |x-2| - |2x+2| \leq 1$
 $\Rightarrow (-\infty, -2] \cup [-\frac{1}{2}, +\infty)$
 $= \mathbb{R} - (-2, -\frac{1}{2})$

ه) $y = \begin{cases} 3 & x \in \mathbb{Z} \\ 2 & x \notin \mathbb{Z} \end{cases} \quad R = \{2, 3\}$



و) $R = [-2, 0)$

$y = \begin{cases} 3x+2 & -2 < x < -1 \\ 3x+1 & -1 < x < 0 \\ 3x & 0 \leq x < 1 \\ 3x-1 & 1 \leq x < 2 \\ 2 & x=2 \end{cases}$



ز) $\rightarrow \sin x = \frac{-b}{ra} = \frac{1}{2} \rightarrow y = \frac{\pi}{6} \text{ min}$
 $\sin x = 1 \rightarrow y = 1$
 $\sin x = -1 \rightarrow y = \pi \text{ max}$

$\Downarrow$
 $R = [\frac{\pi}{6}, \pi]$

ح) $\rightarrow \sin^2 x = \frac{-b}{ra} = -\frac{1}{2}$
 $\sin^2 x = 0 \rightarrow y = 1 \text{ min}$
 $\sin^2 x = 1 \rightarrow y = \pi \text{ max}$

$\Downarrow$
 $R = [1, \pi]$