نام و نام خانوادگی ......آترین جلیلی...... پاسخنامه تشریحی تکلیف شماره ...۳.... کلاس ..دوازدهم سپهر ۶......

$y=\dfrac{x^2+x+2}{x-1} \xrightarrow{x\neq 1} yx-y=x^2+x+2 \rightarrow x^2+x(1-y)+(2+y)=0$

$\rightarrow x=\dfrac{-(1-y)\pm\sqrt{(1-y)^2-4(2+y)}}{2} \Rightarrow \Delta\geq 0 \rightarrow 1+y^2-2y-8-4y\geq 0$ ۱

$\rightarrow y^2-6y-7\geq 0 \rightarrow (y-7)(y+1)\geq 0 \quad \begin{array}{c|ccccc} y & & -1 & & 7 & \\ \hline & + & 0 & - & 0 & + \end{array} \rightarrow R_f=(-\infty,-1]\cup[7,+\infty)$

الف) $y=2x^2-5x+1 \xrightarrow{a>0} x_{min}=\dfrac{-b}{2a}=\dfrac{5}{4} \rightarrow y_{min}=\dfrac{-17}{8} \rightarrow R_f=[\dfrac{-17}{8},+\infty)$

ب) $y=-\sqrt{-x^2+6x-5} \xrightarrow{a<0} x_{max}=\dfrac{-b}{2a}=3 \ ,\ y_{max}=4 \rightarrow R_f=\sqrt{(-\infty,4]}=[0,2]$

ج) $y=-\sqrt{x^3-3x^2+4x+1} \qquad R_f=[0,+\infty)=\sqrt{\mathbb{R}}$ ۲

د) $\log(x^3-6x^2+7x+4) \qquad R_f=(-\infty,+\infty)$

الف) $y=\dfrac{3x+1}{2x-6} \Rightarrow R_f=\mathbb{R}-\{\dfrac{3}{2}\}$

ب) $y=\sqrt{\dfrac{4x+1}{x-2}} \Rightarrow R_f=[0,+\infty)-\{2\}$

$R_f=[\dfrac{4}{\sqrt{3}},+\infty)$

ج) $y=\dfrac{x^2+4}{\sqrt{x^2+3}}=\dfrac{x^2+3+1}{\sqrt{x^2+3}}=\sqrt{x^2+3}+\dfrac{1}{\sqrt{x^2+3}} \Rightarrow$ کمترین مقدار $\sqrt{x^2+3}=\sqrt{3} \rightarrow \sqrt{3}+\dfrac{1}{\sqrt{3}}=\dfrac{4}{\sqrt{3}}$ ۳

د) $y=x^2+\dfrac{1}{x^2+4}=x^2+4+\dfrac{1}{x^2+4}-4 \rightarrow y_{min}=4+\dfrac{1}{4}-4=\dfrac{1}{4} \Rightarrow R_f=[\dfrac{1}{4},+\infty)$

روش اصلی: $y=\dfrac{3x^2+4}{x^2-1} \Rightarrow yx^2-y=3x^2+4 \rightarrow x^2(3-y)+(4+y)=0 \Rightarrow x=\dfrac{0\pm\sqrt{0-4(3-y)(4+y)}}{2(3-y)}$

$\rightarrow 2(3-y)\neq 0 \rightarrow \boxed{y\neq 3} \ ,\ -4(3-y)(4+y)\geq 0 \quad \begin{array}{c|ccccc} y & & -4 & & 3 & \\ \hline & + & 0 & - & \emptyset & + \end{array} \Rightarrow R_f=(-\infty,-4]\cup(3,+\infty)$

روش ۲: $y=\dfrac{3(x^2)+4}{1(x^2)-1} \qquad x^2-1=0 \longrightarrow$ برد خارج بازه ۴

$\Longrightarrow R_f=(-\infty,-4]\cup(3,+\infty)$

x^2 دو سر $\Rightarrow [0,+\infty) \rightarrow \begin{cases} x^2=0 \rightarrow y=-4 \\ x^2\rightarrow+\infty \rightarrow \lim\limits_{x^2\to+\infty}\dfrac{3x^2+4}{x^2-1}=3 \end{cases}$ شامل خود ۳ نمی‌شود

$y=\dfrac{2\sin x-1}{\sin x+3}$ روش اصلی: $y\sin x+3y=2\sin x-1 \rightarrow (y-2)\sin x=-(1+3y) \Rightarrow \sin x=-\dfrac{(1+3y)}{y-2}$

$\rightarrow -1\leq -\dfrac{(1+3y)}{y-2}\leq 1 \longrightarrow -1\leq -\dfrac{(1+3y)}{y-2} \rightarrow -\dfrac{(1+3y)}{y-2}+1\geq 0 \rightarrow \dfrac{-2y-3}{y-2}\geq 0 \quad \begin{array}{ccccc} & -\frac{3}{2} & & 2 & \\ \hline - & 0 & + & \emptyset & - \end{array}$

$\rightarrow \dfrac{-(1+3y)}{y-2}\leq 1 \rightarrow \dfrac{-4y+1}{y-2}\leq 0 \rightarrow \begin{array}{c|ccccc} y & & \frac{1}{4} & & 2 & \\ \hline & - & 0 & + & \emptyset & - \end{array} \Rightarrow y_2\in(-\infty,\dfrac{1}{4}]\cup(2,+\infty) \ ,\ \Rightarrow y_1\in[-\dfrac{3}{2},2)$ ۵

$R_f=y_1\cap y_2=[-\dfrac{3}{2},\dfrac{1}{4}]$

روش ۲: $-1\leq\sin x\leq 1$ $\qquad$ $\sin x+3\neq 0$ مخرج صفر نمی‌شود پس برد داخل بازه است

$\hookrightarrow \sin x=1 \rightarrow y=\dfrac{2(1)-1}{(1)+3}=\dfrac{1}{4} \ ,\ \sin x=-1 \rightarrow y=-\dfrac{3}{2} \rightarrow R_f=[-\dfrac{3}{2},\dfrac{1}{4}]$

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$$f(x) = \frac{x^2 - x + \frac{1}{4}}{x^2 - x + 1}$$

$x^2 - x + 1 \neq 0 \quad \Delta < 0 \rightarrow$ ریشه حقیقی ندارد $\rightarrow D_f = \mathbb{R}$

$$f(x) = \frac{(x-\frac{1}{2})^2}{(x-\frac{1}{2})^2 + \frac{3}{4}} \xrightarrow{(x-\frac{1}{2})^2 = t} f(x) = \frac{t}{t+\frac{3}{4}} \rightarrow R_f = [0,1)$$

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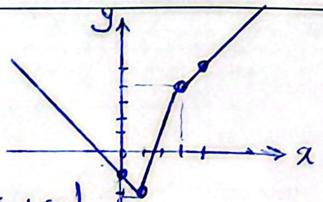
الف) $y = |2x-2| - |x-3| \Rightarrow y = \begin{cases} x+1 & x \geq 3 \\ 3x-5 & 1 \leq x < 3 \\ -x-1 & x < 1 \end{cases}$

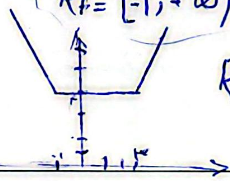
$\begin{vmatrix} 3 \\ 4 \end{vmatrix} , \begin{vmatrix} 1 \\ 5 \end{vmatrix} , \begin{vmatrix} 0 \\ -1 \end{vmatrix} , \begin{vmatrix} 1 \\ -2 \end{vmatrix}$



$R_f = [-2, +\infty)$

ب) $y = |x-3| + |x+1|$ تابع گلدانی

$y = \begin{cases} 2x-2 & x \geq 3 \\ 4 & -1 \leq x \leq 3 \\ -2x+2 & x \leq -1 \end{cases}$ $\quad \begin{vmatrix} 3 \\ 4 \end{vmatrix} \begin{vmatrix} 4 \\ 6 \end{vmatrix} \quad \begin{vmatrix} -1 \\ 4 \end{vmatrix} \begin{vmatrix} -2 \\ 6 \end{vmatrix}$

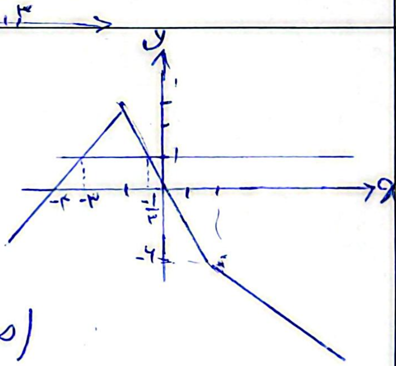


$R_f = [4, +\infty)$

۸

$|x-2| - |2x+2| \leq 1$

$x<-1$	$-1 \leq x < 2$	$x \geq 2$
$-x+2+2x+2 \leq 1$	$-x+2-2x-2 \leq 1$	$x-2-2x-2 \leq 1$
$\rightarrow x \leq -3$ ✓	$\rightarrow x \geq -\frac{1}{3}$ ✓	$\rightarrow x \geq -5$ ✓
	$\rightarrow -\frac{1}{3} \leq x < 2$	$\rightarrow x \geq 2$



$\Rightarrow x \leq -3$ یا $x \geq -\frac{1}{3} \rightarrow x \in (-\infty, -3] \cup [-\frac{1}{3}, +\infty)$

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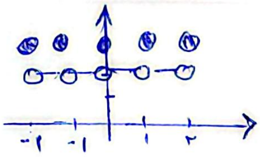
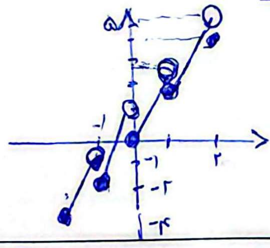
الف) $y = [x] + [3-x] = \underbrace{[x] + [-x]}_{0 \text{ یا } -1} + 3 \rightarrow R_f = \{2, 3\}$



ب) $y = 3x - [x]$

نمودار را به صورت بازه بازه رسم می‌کنیم

$R_f = [-4, 5)$



$\begin{vmatrix} 2 \\ 4 \end{vmatrix} \quad \begin{vmatrix} -2 \\ -4 \end{vmatrix}$

۱۰

الف) $y = \sin^2 x - \sin x + 1 \rightarrow \sin x = \frac{-b}{2a} = \frac{1}{2}, \ f(\frac{1}{2}) = \frac{3}{4} \ min$

$\sin x = 1 \rightarrow y = 1 \qquad \sin x = -1 \rightarrow y = \underset{max}{3} \rightarrow R_f = [\frac{3}{4}, 3]$

ب) $y = \sin^4 x + \sin^2 x + 1 = (\sin^2 x)^2 + \sin^2 x + 1$

$\Rightarrow \sin^2 x = \frac{-b}{2a} = \frac{-1}{2} \rightarrow X \qquad \sin^2 x = 0 \rightarrow y = 1 \ min \qquad \Rightarrow R_f = [1, 3]$

$\sin^2 x = 1 \rightarrow y = 3 \ max$