

$$y = \frac{x^2 + 2}{x-1} \Rightarrow yx - y = x^2 + 2 \Rightarrow x^2 + (1-y)x + 2+y = 0 \Rightarrow \Delta = (y-1)^2 - 4(2+y) \geq 0 \Rightarrow y^2 - 4y - 7 \geq 0 \Rightarrow (y-7)(y+1) \geq 0$$

$P_f = (-\infty, 1] \cup [7, +\infty)$ ✓

الف) $x_0 = x_1 \Rightarrow y = \frac{10}{1} - \frac{20}{2} + 1 = -\frac{10}{1} \Rightarrow P_f = [-\frac{10}{1}, +\infty)$ ✓

ب) $P_f = [0, 2] \Rightarrow P_f = (-\infty, 2] \cap (-\infty, 0] = (-\infty, 0]$ در این حالت

ج) $P_f = [0, +\infty)$ زیرا برای $x \in P_f$ داریم $x^2 + 2 \geq 0$

د) $P_f = \mathbb{R}$ زیرا برای هر $x \in \mathbb{R}$ داریم $x^2 + 2 \geq 0$

الف) $P_f = \mathbb{R} - \{x_0\}$ ✓

ب) $P_f = \mathbb{R} - \{x_0\} \Rightarrow P_f = [0, +\infty) - \{x_0\}$ ✓

ج) $\frac{x^2 + 2}{\sqrt{x^2 + 2}} \geq \frac{1}{\sqrt{x^2 + 2}} \Rightarrow x^2 + 2 \geq 1 \Rightarrow x^2 \geq -1$ همیشه برقرار است $\Rightarrow P_f = [0, +\infty)$

د) $x^2 + 2 \geq \frac{1}{x^2 + 2} \Rightarrow x^2 + 2 \geq 1 \Rightarrow x^2 \geq -1$ همیشه برقرار است $\Rightarrow P_f = [0, +\infty)$

الف) $y = \frac{x^2 + 2}{x-1} \Rightarrow yx - y = x^2 + 2 \Rightarrow x^2 + (1-y)x + 2+y = 0 \Rightarrow \Delta = (y-1)^2 - 4(2+y) \geq 0 \Rightarrow y^2 - 4y - 7 \geq 0 \Rightarrow (y-7)(y+1) \geq 0$

$\Rightarrow x = \frac{y-1 \pm \sqrt{y^2 - 4y - 7}}{2} \geq 0 \Rightarrow \frac{y-1 \pm \sqrt{y^2 - 4y - 7}}{2} \geq 0 \Rightarrow P_f = (-\infty, -1] \cup [7, +\infty)$

ب) $P_f = (-\infty, -1] \cup [7, +\infty)$ زیرا برای $x \in P_f$ داریم $x^2 + 2 \geq 0$

ج) $y = \frac{x^2 + 2}{x-1} \Rightarrow yx - y = x^2 + 2 \Rightarrow x^2 + (1-y)x + 2+y = 0 \Rightarrow \Delta = (y-1)^2 - 4(2+y) \geq 0 \Rightarrow y^2 - 4y - 7 \geq 0 \Rightarrow (y-7)(y+1) \geq 0$

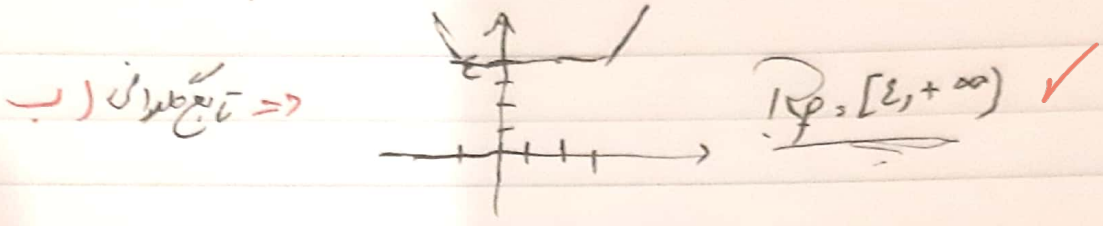
د) $y = \frac{x^2 + 2}{x-1} \Rightarrow yx - y = x^2 + 2 \Rightarrow x^2 + (1-y)x + 2+y = 0 \Rightarrow \Delta = (y-1)^2 - 4(2+y) \geq 0 \Rightarrow y^2 - 4y - 7 \geq 0 \Rightarrow (y-7)(y+1) \geq 0$

ماملو بنیوس

دسته ۱ - $\lim_{n \rightarrow \infty} x_n = 1 \Rightarrow f_{\lim} = 1 \Rightarrow R_f = [-1, 1]$
 $\lim_{n \rightarrow \infty} x_n = 1 \Rightarrow f_{\lim} = 1/2 \quad (\lim_{n \rightarrow \infty} x_n \neq 1)$

$x_n^2 - n + 1 \rightarrow 0 \Rightarrow n \rightarrow \infty \Rightarrow \frac{1}{n} \rightarrow 0 \Rightarrow R_f = [0, 1] \checkmark$
 $-1/2 < x_n^2 - n < +\infty \Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n+1} = 0 \Rightarrow [0, 1] = R_f \checkmark$

الف) $x < 1 \Rightarrow 1 - n + n - 1 = 0 \Rightarrow R = [-1, +\infty)$
 $1 < x < \infty \Rightarrow 1 - n - 1 + n - 1 = -1 \Rightarrow R = (-1, 1)$
 $x < n \Rightarrow 1 - n - 1 + n + 1 = 1 \Rightarrow R = (1, +\infty)$



$x < -1 \Rightarrow 1 - n + n + 1 < 1 \Rightarrow x < -1 \checkmark$
 $-1 < x < 1 \Rightarrow 1 - n - 1 + n - 1 < 1 \Rightarrow -1 < x < 1 \checkmark$
 $1 < x \Rightarrow 1 - n - 1 + n - 1 < 1 \Rightarrow x > 1 \checkmark$
 اجتماع اینها $= R_f = (-\infty, -1] \cup [-1, 1] \cup [1, +\infty) = R = [-1, 1]$

الف) $[n] + [-n] + 1$ $R = \{0, 1\} \checkmark$

ب) $f_n(x) = [x] - 1$ $\Rightarrow$ توافق $f_n(x) + 1$
 $-1 < x < 0 \Rightarrow [x] = -1 \Rightarrow f_n(x) = -2$
 $0 < x < 1 \Rightarrow [x] = 0 \Rightarrow f_n(x) = -1$
 $1 < x < 2 \Rightarrow [x] = 1 \Rightarrow f_n(x) = 0$
 $R_f = [-2, +\infty) = \{(-2, -1) \cup (-1, 0) \cup [0, 1) \cup [1, 2) \cup [2, 3) \dots\}$

الف) $-1/2 \leq \lim_{n \rightarrow \infty} x_n \leq 1/2 \Rightarrow f_{\lim} = 1/2 = \min \Rightarrow R = [1/2, 1]$
 $\lim_{n \rightarrow \infty} x_n = 0 \Rightarrow y = 1$
 ب) $-1/2 \leq \lim_{n \rightarrow \infty} x_n \leq 1/2 \Rightarrow \max \lim_{n \rightarrow \infty} x_n = 1/2 \Rightarrow R = [1/2, 1]$