

$$yx - y = x^2 + x + 1 \Rightarrow x^2 + (1-y)x + y + 1$$

$$\Delta \geq 0 \Rightarrow b^2 - 4ac \geq 0 \Rightarrow (1-y)^2 - 4(y+1) \geq 0 \Rightarrow y^2 - 2y + 1 - 4y - 4 \geq 0$$

$$\Rightarrow y^2 - 4y - 3 \geq 0$$

$$\frac{-1 \pm \sqrt{1+12}}{2} \Rightarrow R_F = (-\infty, -1] \cup [3, +\infty)$$

الف) $\frac{-b}{2a} = \frac{0}{2} \Rightarrow 1(\frac{0}{2})^2 - 0(\frac{0}{2}) + 1 = \frac{-1}{1} \Rightarrow R_F = [-\frac{1}{1}, +\infty)$

ب) $\frac{-b}{2a} = 3 \Rightarrow \sqrt{-1+12-0} = \sqrt{11} = 1 \Rightarrow R_F = [0, 1]$

ج) $\Rightarrow R_F = [0, +\infty)$

د) $\Rightarrow R_F = (-\infty, +\infty)$

الف) $R_F = R - [\frac{0}{2}] \Rightarrow R_F = R - [\frac{0}{2}]$

ب) $R_F(\frac{5x+1}{x-1}) = [0, +\infty) - [\frac{5}{1}] \Rightarrow R_F \sqrt{\frac{5x+1}{x-1}} = \sqrt{[0, +\infty)} - \frac{5}{1} \Rightarrow R_F = [0, +\infty) - [5]$

ج) $R_F \rightarrow x^2 \rightarrow \frac{1}{\sqrt{13}} = \frac{5\sqrt{13}}{13} \Rightarrow R_F = [\frac{5\sqrt{13}}{13}, +\infty)$

د) $y + \varepsilon = x^2 + \varepsilon + \frac{1}{x^2 + \varepsilon} \xrightarrow{x \rightarrow 0} y + \varepsilon = \varepsilon \frac{1}{\varepsilon} \Rightarrow R_F = [\frac{1}{\varepsilon}, +\infty)$

$$yx^2 - y = 3x^2 + \varepsilon \Rightarrow yx^2 - 3x^2 = y + \varepsilon \Rightarrow x^2 = \frac{y + \varepsilon}{y - 3}$$

$$x = \sqrt{\frac{y + \varepsilon}{y - 3}} \Rightarrow R_F = (-\infty, -\varepsilon] \cup (3, +\infty)$$

$x^2 = 0 \rightarrow y = \frac{0 + \varepsilon}{0 - 3} = -\varepsilon$, $x \rightarrow \infty \Rightarrow y \approx 3$, $\frac{y}{3}$

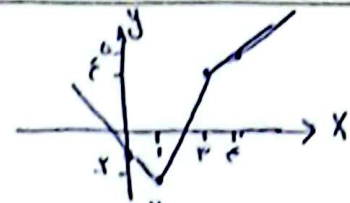
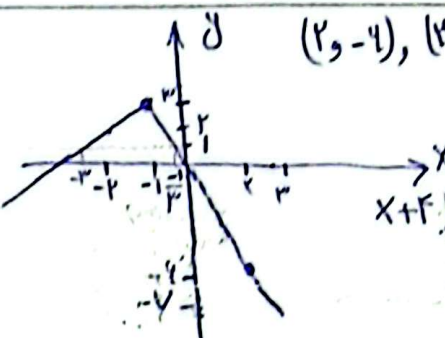
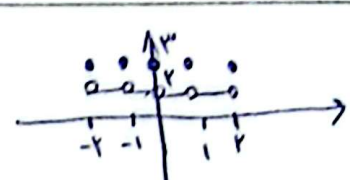
$$\Rightarrow R_F = (-\infty, -\varepsilon] \cup (3, +\infty)$$

$$y \sin x + 3y = 2 \sin x - 1 \rightarrow y \sin x - 2 \sin x = -3y - 1 \rightarrow \sin x = \frac{-3y - 1}{y - 2}$$

$$\Rightarrow -1 \leq \frac{-3y - 1}{y - 2} \leq 1 \Rightarrow R_F = [-\frac{3}{2}, \frac{1}{\varepsilon}] \xrightarrow{\text{محدود}} R_F = R$$

$-1 \leq \sin x \leq 1 \Rightarrow y_1 = \frac{|x| - 1}{1 + 3} = \frac{1}{4}$, $y_2 = \frac{2x(-1) - 1}{-1 + 3} = \frac{3}{2}$

$\Rightarrow R_F = [-\frac{3}{2}, \frac{1}{4}]$

$f(x) = \frac{(x - \frac{1}{x})^2}{(x - \frac{1}{x})^2 + \frac{x}{x}}$ $\xrightarrow{(x - \frac{1}{x}) \rightarrow t} f(x) = \frac{t^2}{t^2 + \frac{x}{x}}$ $(x - \frac{1}{x})^2 + \frac{x}{x} \rightarrow \infty \Rightarrow \frac{0}{\infty} \Rightarrow \frac{0}{\infty} = 0$ $t = 0 \Rightarrow y_1 = 0, t = 0 \Rightarrow y_2 = 1$ $R_f = (-\infty, 0] \cup (1, +\infty)$	6
الف)  $(-2, -1), (1, -2), (3, 2), (4, 4)$ $D_f = R, R_f = [-2, +\infty)$ ب)  $D_f = R, R_f = [0, +\infty)$	7
 $(-2, -1), (-1, -1), (1, 3), (2, 2)$ $x + \epsilon > 1 \Rightarrow x > -3, 1 > -3x \Rightarrow \frac{1}{3} > x$ $\hookrightarrow \text{جواب} = R - (-\frac{1}{3}, -3)$	8
الف)  ب) 	9
الف) $\sin x \begin{cases} 1 \Rightarrow 1 - 1 + 1 = 1 \\ -1 \Rightarrow 1 - (-1) + 1 = 3 \end{cases} \Rightarrow R_f = [\frac{3}{e}, 3]$ $\frac{-b}{\gamma a} = \frac{1}{\gamma} \Rightarrow \frac{1}{e} - \frac{1}{\gamma} + 1 = \frac{3}{e}$ ب) $\sin x \begin{cases} 1 \Rightarrow 1 + 1 + 1 = 3 \\ 0 \Rightarrow 0 + 0 + 1 = 1 \end{cases} \Rightarrow R_f = [\frac{3}{e}, 3]$ $\frac{-b}{\gamma a} = \frac{-1}{\gamma} \Rightarrow (\frac{-1}{\gamma})^2 + \frac{-1}{\gamma} + 1 = \frac{3}{e}$	10