

$$y = \frac{x^2 + x + 1}{x - 1} \Rightarrow yx - y + x^2 + x + 1 = 0 \Rightarrow x^2 + (y+1)x + (y+1) = 0$$

$$\Delta = (y+1)^2 - 4(y+1) \geq 0 \Rightarrow y^2 - 2y - 3 \geq 0 \Rightarrow (y+1)(y-3) \geq 0$$

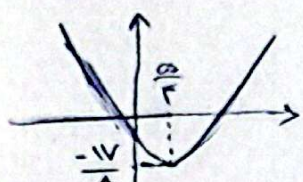
$$\Rightarrow R = [-1, 3]$$

$(-1, 3)$

الف) $y = \frac{x^2 - 5x + 1}{x - 1}$

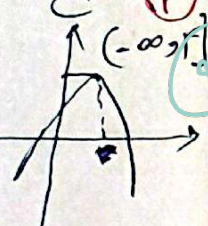
$$\frac{-b}{2a} = \frac{5}{2} = 2.5$$

$$\Rightarrow y = \frac{17}{2}$$

$$R_f = [\frac{17}{2}, \infty)$$


ب) $y = \frac{x^2 - 4x - 5}{x - 1}$

$$\frac{-b}{2a} = \frac{4}{2} = 2 \Rightarrow f(2) = \frac{4 - 8 - 5}{2 - 1} = -9$$

$$R_f = (-\infty, -9]$$


ج) $y = \frac{x^2 - 3x + 1}{x - 1}$

$$\Rightarrow R_f = \mathbb{R} - \{1\}$$

د) $\log \frac{5x^2 - 4x + 1}{x - 1}$

$$\Rightarrow R_f = \mathbb{R}$$

الف) $y = \frac{x^2 + 1}{x - 1}$

$$\Rightarrow R_f = \mathbb{R} - \{1\}$$

ب) $y = \frac{x^2 + 1}{x - 1}$

$$\Rightarrow R_f = \mathbb{R} - \{1\}$$

ج) $\frac{x^2 + 1}{\sqrt{x^2 + 3}}$

$$\sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} = \sqrt{3} + \frac{1}{\sqrt{3}}$$

د) $y = \frac{x^2 + 1}{x^2 + 4}$

$$f = \frac{1}{4} + \frac{17}{4} - 4 = \frac{1}{4} \Rightarrow R_f = [\frac{1}{4}, \infty)$$

ه) $\frac{x^2 + 1}{\sqrt{x^2 + 3}}$

$$\Rightarrow R_f = [\frac{4\sqrt{3}}{3}, \infty)$$

الف) $y = \frac{3x^2 + 4}{x^2 - 1}$

$$\Rightarrow x = \sqrt{\frac{y+4}{y-3}} \Rightarrow \frac{y+4}{y-3} \geq 0 \Rightarrow R_f = (-\infty, -4] \cup (3, \infty)$$

ب) $y = \frac{3\infty + 4}{\infty - 1} = 3$ و $y = \frac{4}{-1} = -4$

کمترین مقدار و بزرگترین مقدار $R_f = (-\infty, -4] \cup (3, \infty)$

(د) $y = \frac{\sin a - 1}{\sin a + 1} \Rightarrow y \sin a + 1 = \sin a + y - y \sin a + 1 \Rightarrow (1-y)(\sin a) = y + 1$
 $\sin a = \frac{y+1}{1-y}$

(1) $\Rightarrow \frac{1-y}{1+y} < 1 \Rightarrow \frac{1-y}{1+y} < \frac{1-y}{1-y} \Rightarrow 1 < 1+y \Rightarrow y > 0$
 (2) $\Rightarrow \frac{1-y}{1+y} > 1 \Rightarrow \frac{1-y}{1+y} > \frac{1-y}{1-y} \Rightarrow 1 > 1+y \Rightarrow y < 0$

$\frac{1-y}{1+y} > 1 \Rightarrow \frac{1-y}{1+y} > \frac{1-y}{1-y} \Rightarrow 1 > 1+y \Rightarrow y < 0$

(ب) $\frac{1-y}{1+y} > 1 \Rightarrow \frac{1-y}{1+y} > \frac{1-y}{1-y} \Rightarrow 1 > 1+y \Rightarrow y < 0$

(ج) $y = \frac{a^2 - a + 1}{a^2 - a + 1} \Rightarrow a^2 - a + 1 \neq 0, \sqrt{b^2 - 4ac} = \sqrt{1 - 4}$
 $\Rightarrow \frac{1}{4} = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{-3}}{2}$
 $\Rightarrow \frac{1}{4} = \frac{1 \pm i\sqrt{3}}{2} \Rightarrow y = \frac{1 \pm i\sqrt{3}}{2}$

(د) $f(-1) = -1, f(1) = 1, f(-1) = 1, f(1) = -1$
 $f(-1) = 1, f(1) = -1, f(-1) = -1, f(1) = 1$
 $f(-1) = -1, f(1) = 1, f(-1) = 1, f(1) = -1$

(هـ) $|x-1| - |x+1| < 1 \Rightarrow f(-1) = 1, f(1) = -1, f(-1) = -1, f(1) = 1$
 $f(-1) = 1, f(1) = -1, f(-1) = -1, f(1) = 1$
 $f(-1) = -1, f(1) = 1, f(-1) = 1, f(1) = -1$

(و) $y = \sin a - \sin a + 1$
 $\frac{-b}{a} = f\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{1}{2} = 0 \Rightarrow y = 0$
 $\sin a + \sin a + 1 = 1 + 1 = 2$
 $\Rightarrow Rf = [1, 2]$