

$$y = \frac{x^2 + x + 2}{x - 1}$$

$$yx - y = x^2 + x + 2 \rightarrow x^2 + (1 - y)x + y + 2 = 0$$

$$\Delta = b^2 - 4ac = (1 - y)^2 - 4(y + 2) = y^2 - 4y - 7$$

$$x = \frac{y - 1 \pm \sqrt{y^2 - 4y - 7}}{2}$$

$$y^2 - 4y - 7 \geq 0 \quad \begin{array}{c} -1 \quad \quad \quad \sqrt{7} \\ + \quad \quad - \quad \quad + \end{array}$$

$$R_f = (-\infty, -1] \cup [\sqrt{7}, +\infty)$$

الف) $\text{ext}_{\min} \left| \begin{array}{c} -\frac{5}{x} \\ \frac{1}{x} \end{array} \right| \quad R_f = [-\frac{1}{x}, +\infty)$

ب) $\text{ext}_{\max} \left| \begin{array}{c} y \\ 4 \end{array} \right| \quad R_f = \sqrt{(-\infty, 4]} = [0, 2]$

ج) $\sqrt{1R} = [0, +\infty)$

د) $R_f = 1R$

الف) $R_f = 1R - \left\{ \frac{3}{y} \right\}$

ب) $R_f = \sqrt{1R - \{4\}} = [0, +\infty) - \{2\}$

ج) $\sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} = y \quad R_f = [\sqrt{3}, +\infty)$

د) $y = x^2 + 4 + \frac{1}{x^2 + 4} - 4 \quad R_f = [4, +\infty) - 4 = [0, +\infty)$

$$\frac{3x^2 + 4}{x^2 - 1} = y \rightarrow x^2 = -\frac{-y - 4}{y - 3} = \frac{y + 4}{y - 3} \rightarrow x = \pm \sqrt{\frac{y + 4}{y - 3}} \quad \begin{array}{c} -4 \quad \quad \quad 3 \\ + \quad \quad - \quad \quad + \end{array}$$

$y = \frac{3x^2 + 4}{x^2 - 1} \rightarrow x = 0 \rightarrow y = -4$
 $\rightarrow x = +\infty \rightarrow y = 3$
 منحنی منفرجه است

$$R_f = (-\infty, -4] \cup (3, +\infty)$$

$$y = \frac{2 \sin x - 1}{\sin x + 3} \rightarrow \sin x = -\frac{3y + 1}{y - 2} = \frac{3y + 1}{2 - y} \rightarrow -1 \leq \frac{3y + 1}{2 - y} \leq 1 \rightarrow -\frac{3}{4} \leq y \leq \frac{1}{4}$$

منحنی منفرجه است
 $\sin x = -1 \rightarrow y = -\frac{3}{4}$

$\sin x = 1 \rightarrow y = \frac{1}{4}$

$$\left[-\frac{3}{4}, \frac{1}{4}\right]$$

$$f(x) = \frac{(x - \frac{1}{4})^2}{(x - \frac{1}{4})^2 + \frac{3}{4}}$$

مربع مقلوب من 3/4

$$(x - \frac{1}{4})^2 \rightarrow \min = 0 \rightarrow y = 0$$

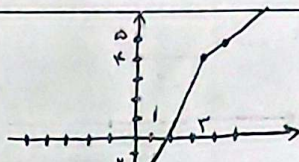
$$\rightarrow \max = +\infty \rightarrow y = 1$$

$$R_f = [0, 1)$$

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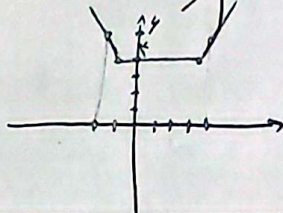
الف) $|2x - 2| + |x - 3| = y$

$$R_f = \mathbb{R}$$



ب) $|x - 3| + |x + 1| = y$

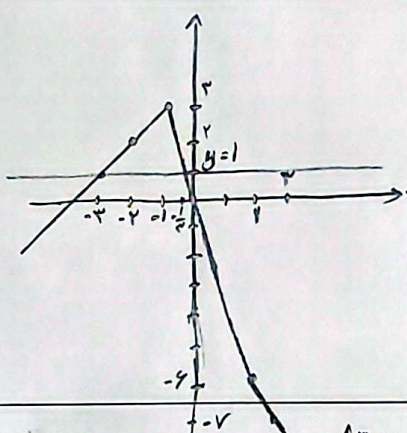
$$R_f = [4, +\infty)$$



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$$|x - 2| - |2x + 2| \leq 1$$

$$x \in \mathbb{R} - [-3, -\frac{1}{4})$$



$$x \leq -1 \rightarrow x + 4 \leq 1 \rightarrow x \leq -3 \checkmark$$

$$-1 \leq x \leq 2 \rightarrow -3x \leq 1 \rightarrow -\frac{1}{3} \leq x \checkmark$$

$$2 \leq x \rightarrow -x - 4 \leq 1 \rightarrow x \leq -5 \times$$

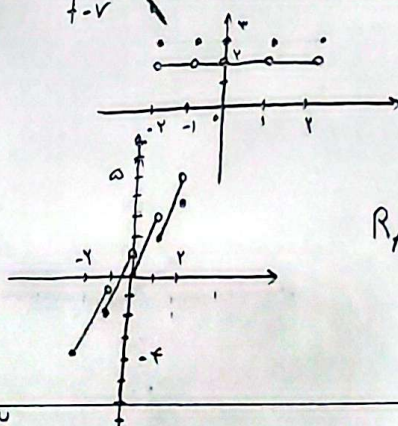
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الف) $f(x) = [x] + [-x] + 3$

$$f(x) \begin{cases} x \in \mathbb{Z} & 3 \\ x \notin \mathbb{Z} & 2 \end{cases}$$

$$R_f = \{2, 3\}$$

ب) $f(x) = 3x - [x]$
 $y' = 3$



$$R_f = [-4, 5)$$

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الف) $\min = -1 \rightarrow y = 3$
 $\max = 1 \rightarrow y = 1$
 $\sin x = -\frac{b}{\sqrt{a}} = \frac{1}{4} \rightarrow y = \frac{3}{4}$

$$R_f = [\frac{3}{4}, 3]$$

ب) $\min = 0 \rightarrow y = 1$
 $\max = 1 \rightarrow y = 3$
 $\sin x = -\frac{b}{\sqrt{a}} = -\frac{1}{4} \in [-1, 1]$

$$R_f = [1, 3]$$

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