

$$y = \frac{x^2 + x + 2}{x - 1} \Rightarrow y(x - 1) = x^2 + x + 2 \Rightarrow x^2 + x(1 - y) + (2 + y) = 0 \Rightarrow x = \frac{-(1 - y) \pm \sqrt{(1 - y)^2 - 4(2 + y)}}{2} \quad (1)$$

$$\Delta \geq 0 \Rightarrow (1 - y)^2 - 4(2 + y) \geq 0 \Rightarrow 1 + y^2 - 2y - 8 - 4y - 4 = y^2 - 6y - 11 \geq 0 \Rightarrow (y - 7)(y + 1) \geq 0 \Rightarrow y \leq -1 \text{ یا } y \geq 7$$

$$R_f = (-\infty, -1] \cup [7, +\infty)$$

$$(2) \quad \text{الف) } y = 2x^2 - 5x + 1 \quad \text{ب) } y = \sqrt{-x^2 + 4x - 5}$$

$$1. \text{ } y = \sqrt{-x^2 + 4x - 5} \Rightarrow y^2 = -x^2 + 4x - 5 \Rightarrow x^2 - 4x + y^2 + 5 = 0$$

$$R = \mathbb{R} = (-\infty, +\infty) \quad \text{چون همیشه مقدار درستی می‌دهد}$$

$$y = \sqrt{-x^2 + 4x - 5} \Rightarrow -x^2 + 4x - 5 \geq 0 \Rightarrow x^2 - 4x + 5 \leq 0 \Rightarrow (x - 2)^2 \geq -1 \Rightarrow x \in \mathbb{R}$$

$$y = 2x^2 - 5x + 1 \Rightarrow \Delta = 25 - 8 = 17 > 0 \Rightarrow x = \frac{5 \pm \sqrt{17}}{4} \Rightarrow R = \left[\frac{17}{4}, +\infty \right)$$

$$(3) \quad \text{الف) } y = \frac{x^2 + 2}{\sqrt{x^2 + 3}} \Rightarrow y\sqrt{x^2 + 3} = x^2 + 2 \Rightarrow y^2(x^2 + 3) = (x^2 + 2)^2 \Rightarrow y^2 x^2 + 3y^2 = x^4 + 4x^2 + 4 \Rightarrow x^4 + (4 - y^2)x^2 + (4 - 3y^2) = 0$$

$$\Rightarrow \sqrt{x^2 + 3} \text{ همیشه مقدار مثبتی است} \Rightarrow \sqrt{x^2 + 3} = \frac{x^2 + 2}{y} \Rightarrow \sqrt{x^2 + 3} = \frac{x^2 + 2}{y}$$

$$R = \left[\frac{\sqrt{3}}{2}, +\infty \right) = \left[\frac{1}{\sqrt{3}}, +\infty \right)$$

$$y = \frac{x^2 + 2}{\sqrt{x^2 + 3}} \Rightarrow y\sqrt{x^2 + 3} = x^2 + 2 \Rightarrow y^2(x^2 + 3) = (x^2 + 2)^2 \Rightarrow x^4 + (4 - y^2)x^2 + (4 - 3y^2) = 0$$

$$(4) \quad y = \frac{3x^2 + 2}{x^2 - 1} \Rightarrow x = \frac{3y^2 + 2}{y^2 - 1} \Rightarrow xy^2 - x = 3y^2 + 2 \Rightarrow xy^2 - 3y^2 = 2 + x \Rightarrow y^2(x - 3) = 2 + x \Rightarrow y^2 = \frac{2 + x}{x - 3}$$

$$\Rightarrow y^2(x - 3) = 2 + x \Rightarrow y^2 = \frac{2 + x}{x - 3} \Rightarrow y = \sqrt{\frac{2 + x}{x - 3}} \Rightarrow R = (-\infty, -\sqrt{\frac{2 + x}{x - 3}}) \cup (\sqrt{\frac{2 + x}{x - 3}}, +\infty)$$

$$y = \frac{3x^2 + 2}{x^2 - 1} \quad x = 0 \Rightarrow y = -2 \quad x = +\infty \Rightarrow y = 3 \Rightarrow R = (-\infty, -2] \cup [3, +\infty)$$

$$(5) \quad y = \frac{2 \sin x - 1}{\sin x + 3} \Rightarrow y(\sin x + 3) = 2 \sin x - 1 \Rightarrow y \sin x + 3y = 2 \sin x - 1 \Rightarrow (y - 2) \sin x = -3y - 1 \Rightarrow \sin x = \frac{-3y - 1}{y - 2}$$

$$\Rightarrow \sin x = \frac{-3y - 1}{y - 2} \Rightarrow \sin x = \frac{1 + 3y}{2 - y} \Rightarrow -1 \leq \frac{1 + 3y}{2 - y} \leq 1 \Rightarrow \begin{cases} 2 - y \leq 1 + 3y \Rightarrow 2y \geq 1 \Rightarrow y \geq \frac{1}{2} \\ 1 + 3y \leq 2 - y \Rightarrow 4y \leq 1 \Rightarrow y \leq \frac{1}{4} \end{cases}$$

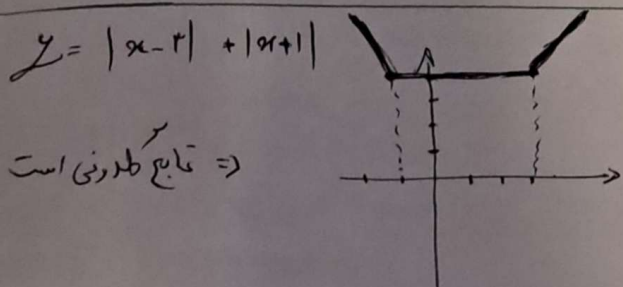
$$R = \left[-\frac{1}{4}, \frac{1}{2} \right]$$

$$y = \frac{2 \sin x - 1}{\sin x + 3} \quad x = 1 \Rightarrow y = \frac{1}{4} \quad x = -1 \Rightarrow y = -\frac{3}{2} \Rightarrow R = \left[-\frac{3}{2}, \frac{1}{4} \right]$$

$$(6) \quad f(x) = \frac{x^2 - x + 1}{x^2 - x + 1} \quad x^2 - x + 1 \neq 0 \Rightarrow x \in \mathbb{R} \quad x^2 - x = -\frac{1}{x} \Rightarrow y = 0$$

$$R = [0, 1)$$

$$x^2 - x = +\infty \Rightarrow y = 1$$

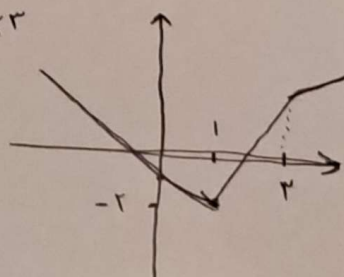


$\Rightarrow$ تابع مذبذب است

$$(7) \quad \text{الف) } y = |2x - 1| - |x - 3|$$

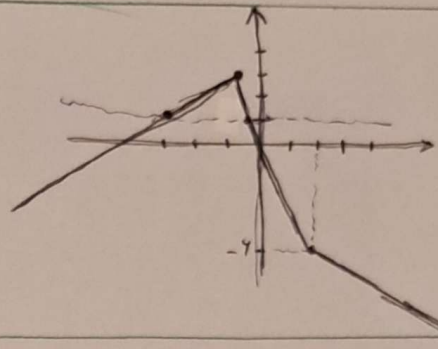
$$\begin{cases} x + 1 & x \geq 3 \\ 3x - 5 & 1 < x < 3 \\ -x - 1 & x < 1 \end{cases}$$

$$R = [-2, +\infty)$$



$$|x-2| - |2x+2| \leq 1$$

$$\begin{cases} -x-4 & x \geq 2 \\ -3x & -1 \leq x \leq 2 \\ x+4 & x \leq -1 \end{cases}$$

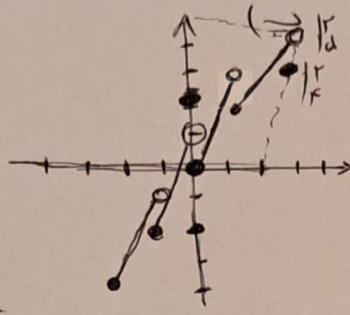


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$$\left(-\infty, -\frac{1}{3} \right] \cup \left[-\frac{1}{3}, +\infty \right)$$

$$y = 3x - [x]$$

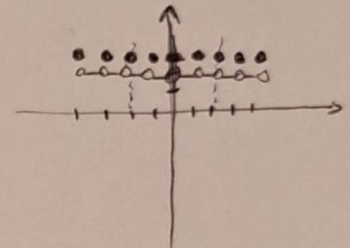
$$\begin{cases} -2 \leq x < -1 & y = 3x+2 \\ -1 \leq x < 0 & y = 3x+1 \\ 0 \leq x < 1 & y = 3x \\ 1 \leq x < 2 & y = 3x-1 \\ x = 2 & y = 5 \end{cases}$$



$$R = [-4, \infty)$$

$$y = [x] + [3-x] = [x] + [-x] + 3 \quad \text{الف}$$

$$R = \{2, 3\}$$



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$$y = \sin^2 x + \sin^2 x + 1$$

$$\sin^2 x = -\frac{b}{ra} = -\frac{1}{r} \quad \text{GGE}$$

$$\sin^2 x = 1 \Rightarrow y = 3$$

$$\sin^2 x = 0 \Rightarrow y = 1$$

$$R = [1, 3]$$

ب.

$$y = \sin^2 x - \sin x + 1$$

$$\sin x = -\frac{b}{ra} = \frac{1}{r} \Rightarrow y = \frac{r}{r}$$

$$\sin x = 1 \Rightarrow y = 1$$

$$\sin x = -1 \Rightarrow y = 3$$

$$R = \left[\frac{r}{r}, 3 \right]$$

الف

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