

19.10

$$x^2 + x + 2 = yx - y \rightarrow x^2 + (1-y)x + (2+y) = 0 \quad x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$1 + y^2 - 2y = 1 - 4y \geq 0 \rightarrow y^2 - 4y - 1 \geq 0 \rightarrow (y-2)(y+1) \geq 0$$

$$R_f = (-\infty, -1] \cup [2, +\infty)$$

$$\begin{array}{c|c|c|c} + & - & + & + \\ \hline + & - & + & + \end{array}$$

الف) $a > 0 \left\{ \begin{array}{l} -\frac{b}{2a} = \frac{1}{x} = x_{\min} \\ -\frac{1}{x} = y_{\min} \end{array} \right\} R_f = [-\frac{1}{x}, +\infty)$

ب) $a < 0 \left\{ \begin{array}{l} -\frac{b}{2a} = 2 = x_{\max} \\ x = y_{\max} \end{array} \right\} R_f = (-\infty, 2] = [0, 2]$

ج) $\xrightarrow{\text{کلی دوم}} R_f = [0, +\infty) > \xrightarrow{\text{کلی دوم}} R_f = (-\infty, +\infty)$

الف) $\xrightarrow{\text{کلی سوم}} R_f = R - \left\{ \frac{3}{4} \right\} > \xrightarrow{\text{کثرین مقدار } x^2 \text{ برابر 0 است}} 0 + \frac{1}{x} = \frac{1}{x}$

ب) $\xrightarrow{\text{کلی سوم}} R_f = \sqrt{R - \left\{ \frac{3}{4} \right\}} = [0, +\infty) - \left\{ \frac{3}{4} \right\} \quad R = \left[\frac{1}{x}, +\infty \right)$

ج) $\xrightarrow{\text{کلی مابین}} \frac{x^2 + 3 + 1}{\sqrt{x^2 + 3}} = \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} \rightarrow \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}} \quad R = \left[\frac{4}{\sqrt{3}}, +\infty \right)$

روش ۱) $yx^2 - y = 2x^2 + 4 \rightarrow (2-y)x^2 + (4+y) = 0 \rightarrow (2-y)x^2 = -(4+y)$

$$x^2 = \frac{-y-4}{2-y} \rightarrow x = \pm \sqrt{\frac{-y-4}{2-y}} \rightarrow \begin{array}{c|c|c|c} + & - & + & + \\ \hline + & - & + & + \end{array} \rightarrow R = \begin{array}{l} (-\infty, -4] \cup (2, +\infty) \\ (-\infty, -4] \cup (3, +\infty) \end{array}$$

روش ۲) $x^2 = t \rightarrow \frac{2t-4}{t-1} \quad \left. \begin{array}{l} t=0 \rightarrow 4 \\ t=+\infty \rightarrow 2 \end{array} \right\} R = \begin{array}{l} (-\infty, -4] \cup (2, +\infty) \\ (-\infty, -4] \cup (3, +\infty) \end{array}$

روش ۳) $y \sin x + 2y = 2 \sin x - 1 \rightarrow (2-y) \sin x = 2y + 1 \rightarrow \sin x = \frac{2y+1}{2-y}$

$$\sin x \in [-1, 1] \rightarrow -1 \leq \frac{2y+1}{2-y} \leq 1 \rightarrow \begin{array}{l} -\frac{3}{2} \leq y \\ y \leq \frac{1}{2} \end{array} \quad R = \left[-\frac{3}{2}, \frac{1}{2} \right]$$

روش ۴) $\sin x = t \rightarrow y = \frac{2t-1}{t+2} \quad \left. \begin{array}{l} t=-1 \rightarrow -\frac{3}{2} \\ t=1 \rightarrow \frac{1}{2} \end{array} \right\} R = \left[-\frac{3}{2}, \frac{1}{2} \right]$

$$x^2 - x + 1 \neq 0 \xrightarrow{\Delta < 0} (-1)^2 - (4)(1)(1) = -3 \rightarrow D = R \checkmark$$

$\rightarrow x \neq 0 \quad y \neq 1$

$$y x^2 - y x + y = x^2 - x + \frac{1}{x} \rightarrow (y-1)x^2 + (1-y)x + (y - \frac{1}{x}) = 0 \quad x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

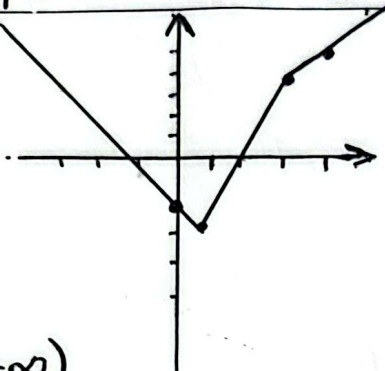
$$y^2 + 1 - 2y - 4y^2 + 4y - 1 \rightarrow -3y^2 + 3y \geq 0 \rightarrow -y^2 + y \geq 0 \quad y(y-1)$$

$\xrightarrow{0 \quad 1} R = [0, 1] \rightarrow R = [0, 1]$

الف)

$$\begin{aligned} x=4 &\rightarrow y=5 \\ x=3 &\rightarrow y=4 \\ x=1 &\rightarrow y=-2 \\ x=0 &\rightarrow y=-1 \end{aligned}$$

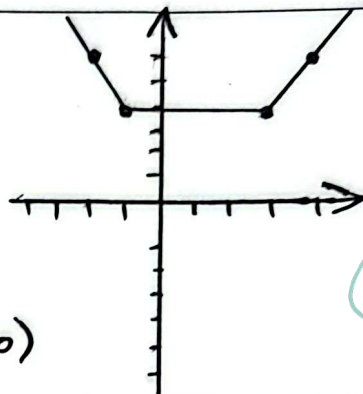
$$R = [-2, +\infty)$$



ب)

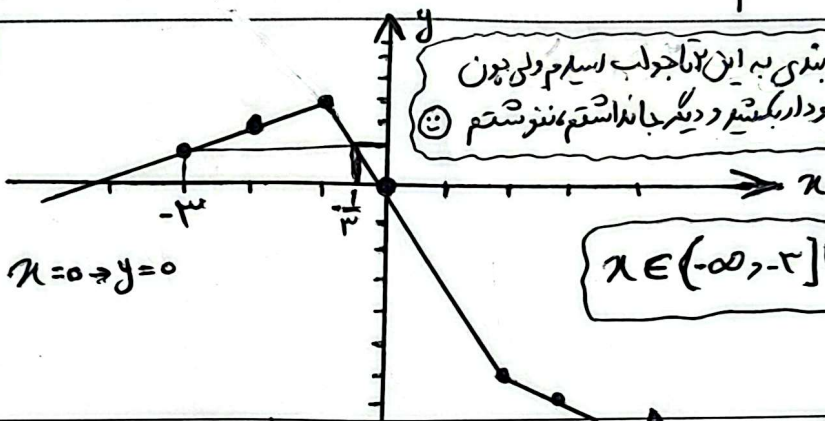
$$\begin{aligned} x=4 &\rightarrow y=6 \\ x=3 &\rightarrow y=4 \\ x=-1 &\rightarrow y=4 \\ x=-2 &\rightarrow y=6 \end{aligned}$$

$$R = [4, +\infty)$$



$$y = \underbrace{|x-2|}_{-1} - \underbrace{|2x+2|}_{-1}$$

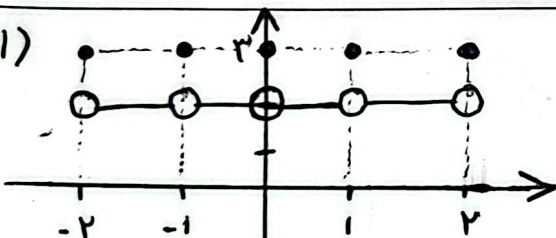
$$\begin{aligned} x=3 &\rightarrow y=-7 \\ x=2 &\rightarrow y=-6 \\ x=-1 &\rightarrow y=3 \\ x=-2 &\rightarrow y=4 \end{aligned}$$



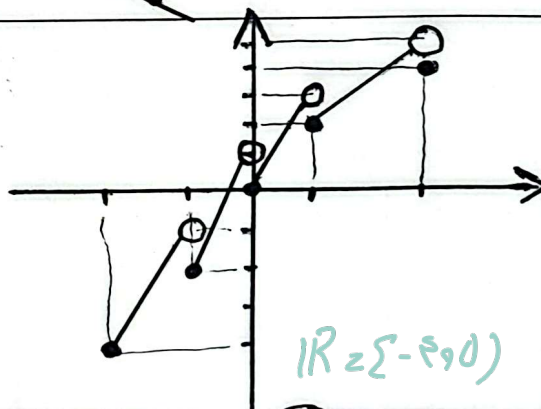
نکته: با بازه بندی به این تابع جواب اسلام ولی چون گفته شد که نمودار یکسره درگیر باشد استم، منفرشته

$$x \in (-\infty, -2] \cup [-\frac{1}{2}, +\infty)$$

الف)



ب)



در مقادیر صحیح برابر ۳ است
در مقادیر دیگر برابر ۲ است

کامل تغییر بازه بندی

$$R = [-2, 3]$$

الف) $\sin x = t$
 $\sin x \in [-1, 1]$

$$t^2 - t + 1$$

$$\begin{aligned} t = -\frac{b}{2a} &\rightarrow y = \left(\frac{3}{4}\right) \\ t = 1 &\rightarrow 1 \\ t = -1 &\rightarrow 3 \end{aligned} \quad \left. \vphantom{\begin{aligned} t = -\frac{b}{2a} &\rightarrow y = \left(\frac{3}{4}\right) \\ t = 1 &\rightarrow 1 \\ t = -1 &\rightarrow 3 \end{aligned}} \right\} \left[\frac{3}{4}, 3 \right]$$

ب) $\sin^2 x = t$
 $\sin^2 x \in [0, 1]$

$$\begin{aligned} t = -\frac{b}{2a} &\rightarrow y = -\frac{1}{2} \rightarrow y \neq 0 \\ t = 0 &\rightarrow 1 \\ t = 1 &\rightarrow 3 \end{aligned} \quad \left. \vphantom{\begin{aligned} t = -\frac{b}{2a} &\rightarrow y = -\frac{1}{2} \rightarrow y \neq 0 \\ t = 0 &\rightarrow 1 \\ t = 1 &\rightarrow 3 \end{aligned}} \right\} [1, 3]$$