

$$x^2 + x + 2 = yx - y \rightarrow x^2 + (1-y)x + (2+y) = 0 \quad x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$1 + y^2 - 2y = 1 - 4y \geq 0 \rightarrow y^2 - 4y - 1 \geq 0 \rightarrow (y-2)(y+1) \geq 0$$

$$R_f = (-\infty, -1] \cup [2, +\infty)$$

$$\begin{array}{c|c|c|c} & -1 & 2 & \\ \hline & + & - & + \end{array}$$

الف) $a > 0 \left\{ \begin{array}{l} -\frac{b}{2a} = \frac{1}{x} = x_{\min} \\ -\frac{1}{x} = y_{\min} \end{array} \right\} R_f = \left[-\frac{1}{x}, +\infty \right)$

ب) $a < 0 \left\{ \begin{array}{l} -\frac{b}{2a} = 2 = x_{\max} \\ x = y_{\max} \end{array} \right\} R_f = \left[-\infty, 2 \right] = [0, 2]$

ج) $\xrightarrow[\text{دوم}]{\text{کلی}} R_f = [0, +\infty) \quad > \xrightarrow[\text{دوم}]{\text{کلی}} R_f = (-\infty, +\infty)$

الف) $\xrightarrow[\text{سوم}]{\text{کلی}} R_f = R - \left\{ \frac{2}{x} \right\} \quad > \xrightarrow[\text{بهرایه 0 است}]{\text{کمترین مقدار}} 0 + \frac{1}{x} = \frac{1}{x} \downarrow$

ب) $\xrightarrow[\text{سوم}]{\text{کلی}} R_f = \sqrt{R - \{2\}} = [0, +\infty) - \{2\} \quad R = \left[\frac{1}{x}, +\infty \right)$

ج) $\xrightarrow[\text{چهارم}]{\text{کلی}} \frac{x^2 + 2 + 1}{\sqrt{x^2 + 2}} = \sqrt{x^2 + 2} + \frac{1}{\sqrt{x^2 + 2}} \rightarrow \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{x}{\sqrt{2}} \downarrow$
 $R = \left[\frac{x}{\sqrt{2}}, +\infty \right)$

روش ۱) $yx^2 - y = 2x^2 + 4 \rightarrow (2-y)x^2 + (4+y) = 0 \rightarrow (2-y)x^2 = -(4+y)$

$x^2 = \frac{-y-4}{2-y} \rightarrow x = \pm \sqrt{\frac{-y-4}{2-y}} \rightarrow \begin{array}{c|c|c|c} & -4 & 2 & \\ \hline & + & - & + \end{array} \rightarrow R = \downarrow$
 $(-\infty, -4] \cup (2, +\infty)$

روش ۲) $\xrightarrow{x^2=t} \frac{2t+4}{t-1} \quad \left. \begin{array}{l} t=0 \rightarrow 4 \\ t=+\infty \rightarrow 2 \end{array} \right\} R =$
 $x^2 \in [0, +\infty) \xrightarrow{x^2 \neq 1} x^2 \in [0, 1) \cup (1, +\infty) \quad \left. \begin{array}{l} t=0 \rightarrow 4 \\ t=+\infty \rightarrow 2 \end{array} \right\} R =$
 $(-\infty, -4] \cup (2, +\infty)$

روش ۳) $y \sin x + 2y = 2 \sin x - 1 \rightarrow (2-y) \sin x = 2y + 1 \rightarrow \sin x = \frac{2y+1}{2-y}$

$\sin x \in [-1, 1] \rightarrow -1 \leq \frac{2y+1}{2-y} \leq 1 \rightarrow \begin{array}{l} -\frac{3}{2} \leq y \\ y \leq \frac{1}{2} \end{array} \left\{ R = \left[-\frac{3}{2}, \frac{1}{2} \right] \right.$

روش ۴) $\xrightarrow{\sin x=t} y = \frac{2t-1}{t+2} \quad \left. \begin{array}{l} t=-1 \rightarrow -\frac{3}{2} \\ t=1 \rightarrow \frac{1}{2} \end{array} \right\} R = \left[-\frac{3}{2}, \frac{1}{2} \right]$
 $\sin x \in [-1, 1]$

$$x^2 - x + 1 \neq 0 \xrightarrow{\Delta < 0} (-1)^2 - (4)(1)(1) = -3 \rightarrow D = R$$

$$y x^2 - y x + y = x^2 - x + \frac{1}{4} \rightarrow (y-1)x^2 + (1-y)x + (y-\frac{1}{4}) = 0 \quad x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

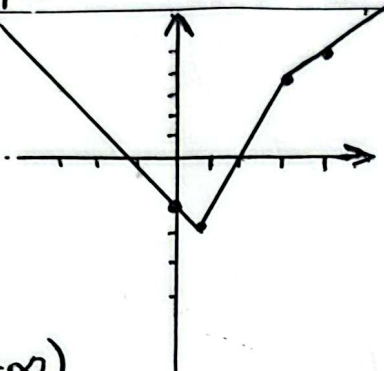
$$y^2 + 1 - 2y - 4y^2 + 4y - 1 \rightarrow -3y^2 + 3y \geq 0 \rightarrow -y^2 + y \geq 0 \quad y(y-1)$$

$$\frac{0}{-1} \frac{1}{1} \rightarrow R = [0, 1]$$

الف)

$$\begin{aligned} x=4 &\rightarrow y=5 \\ x=3 &\rightarrow y=4 \\ x=1 &\rightarrow y=-2 \\ x=0 &\rightarrow y=-1 \end{aligned}$$

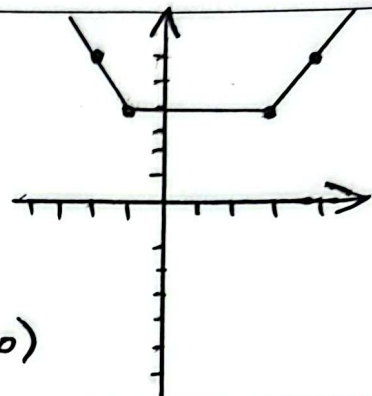
$$R = [-2, +\infty)$$



ب)

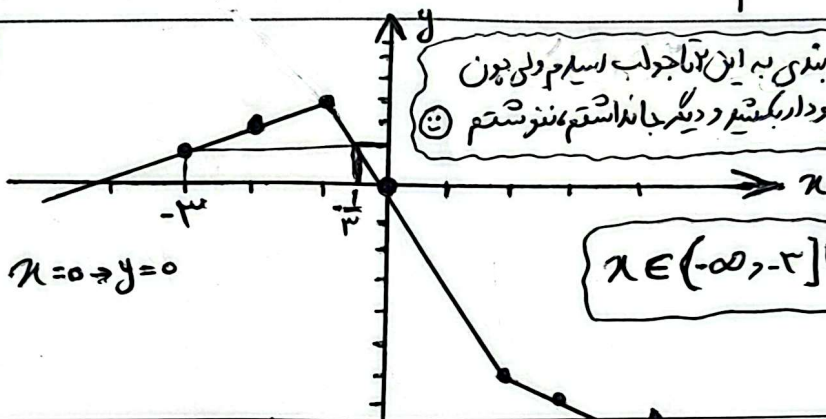
$$\begin{aligned} x=4 &\rightarrow y=6 \\ x=3 &\rightarrow y=4 \\ x=-1 &\rightarrow y=4 \\ x=-2 &\rightarrow y=6 \end{aligned}$$

$$R = [4, +\infty)$$



$$y = \underbrace{|x-2|}_{-1} - \underbrace{|2x+2|}_{-1}$$

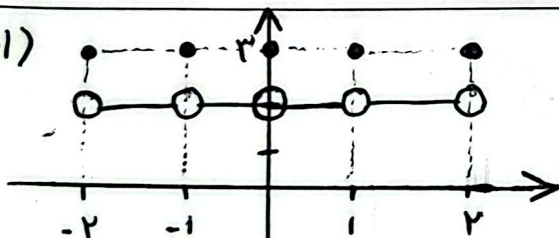
$$\begin{aligned} x=3 &\rightarrow y=-7 \\ x=2 &\rightarrow y=-6 \\ x=-1 &\rightarrow y=3 \\ x=-2 &\rightarrow y=4 \end{aligned}$$



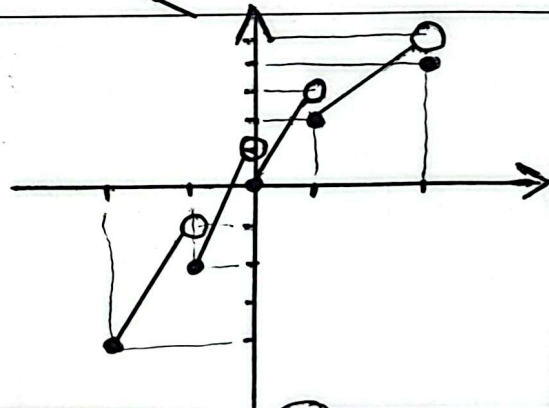
نکته: بابازه بندی به این تابع جواب اسلام ولی چون گفته شد که نمودار یکسره درگیر باشد استقامت منور شستم ☺

$$x \in (-\infty, -2] \cup [-\frac{1}{2}, +\infty)$$

الف)



ب)



در مقادیر صحیح برابر ۳ است
در مقادیر دیگر برابر ۲ است

$$\text{الف) } \sin x = t \rightarrow t^2 - t + 1$$

$$\sin x \in [-1, 1]$$

$$t^2 - t + 1$$

$$\begin{aligned} t = -\frac{b}{2a} &\rightarrow y = \left(\frac{5}{4}\right) \\ t = 1 &\rightarrow 1 \\ t = -1 &\rightarrow 3 \end{aligned}$$

$$\left[\frac{5}{4}, 3\right]$$

$$\text{ب) } \sin^2 x = t \rightarrow t^2 + t + 1$$

$$\sin^2 x \in [0, 1]$$

$$\begin{aligned} t = -\frac{b}{2a} &\rightarrow y = -\frac{1}{4} \rightarrow y = 0 \\ t = 0 &\rightarrow 1 \\ t = 1 &\rightarrow 3 \end{aligned}$$

$$\left[1, 3\right]$$