

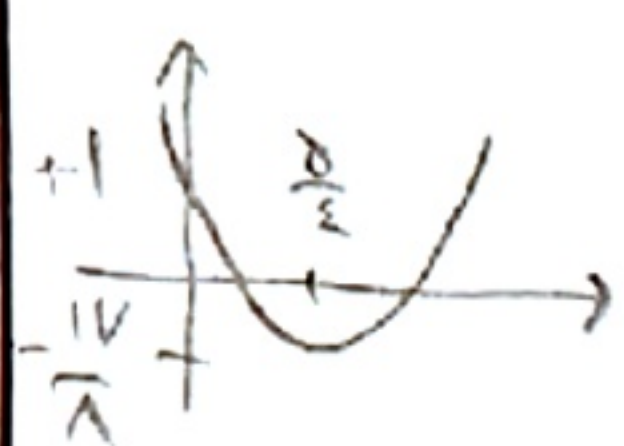
$$y = \frac{x^2 + x + 2}{x - 1} \rightarrow yx - y = x^2 + x + 2 \rightarrow 1 = x^2 + (1 - y)x + 2 + y$$

$$x = \frac{-(1-y) \pm \sqrt{(1-y)^2 - 4(2+y)}}{2} \rightarrow (1-y)^2 - 4(y+2) \geq 0 \rightarrow y^2 - 6y - 7 \geq 0$$

$$y \notin (-1, +7)$$

$$R_f = \mathbb{R} - (-1, +7)$$

الف) $y = 2x^2 - 8x + 1$



$R_f = [-\frac{17}{2}, +\infty)$

ب) $y = \sqrt{-x^2 + 4x - 8}$ $\rightarrow (-\infty, 4]$

$-x^2 + 4x - 8 \geq 0 \rightarrow R = (-\infty, 4] \quad R_f = [0, 2]$

ج) $y = \sqrt{x^2 - 3x^2 + 4x + 1}$

$g(x) = x^2 - 3x^2 + 4x + 1 \rightarrow R_g = \mathbb{R}$

$y = \sqrt{(-\infty, +\infty)} \rightarrow R_f = [0, +\infty)$

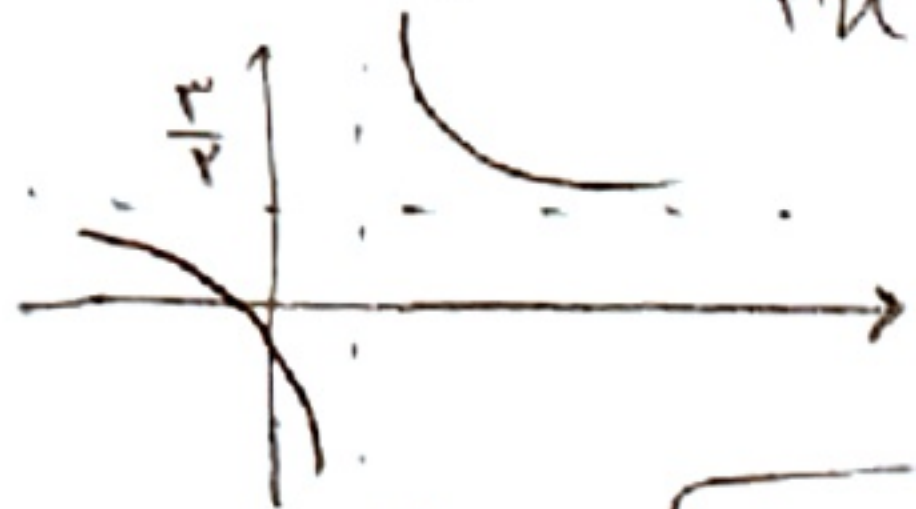
د) $\log(x^2 - 3x^2 + 4x + 1)$

$g(x) = x^2 - 3x^2 + 4x + 1 \rightarrow R_g = (-\infty, +\infty)$

$(-\infty, +\infty) \log \rightarrow R_f = (-\infty, +\infty)$

الف) $y = \frac{2x+1}{x-2}$

$R_f = \mathbb{R} - \{\frac{1}{2}\}$



ج) $y = \frac{x^2 + 4}{\sqrt{x^2 + 3}} = \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}}$

$y_{min} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3} \rightarrow R_f = [\frac{4\sqrt{3}}{3}, +\infty)$

ب) $y = \sqrt{\frac{4x+1}{x-2}}$ $\rightarrow (-\infty, +\infty) \cup \{2\}$

$\frac{4x+1}{x-2} \geq 0 \rightarrow R = \mathbb{R} - \{2\}$

د) $y = x + \frac{1}{x^2 + 4} \rightarrow \frac{x^2}{x^2 + 4} \geq 0$

$x_{min} = 0 \rightarrow y_{min} = \frac{1}{4} \rightarrow R_f = [\frac{1}{4}, +\infty)$

الف) $y = \frac{2x^2 + 4}{x^2 - 1}$

$y \neq 1; x = \frac{0 \pm \sqrt{4(y-2)(y+2)}}{2-y}$

$y = 1; 0 = 4 + y \rightarrow y = -4 \rightarrow 0 \in \mathbb{R}$

$yx^2 - y = 2x^2 + 4 \rightarrow (2-y)x^2 + 4 + y = 0$

$4(y+2)(y-2) \geq 0 \rightarrow y \in \mathbb{R} - (-2, +2]$

$R_f = (-\infty, -2] \cup (2, +\infty)$

ب) $\frac{2x^2 + 4}{x^2 - 1}$

$x^2_{min} = 0 \rightarrow \frac{4}{-1} = -4$
 $x^2_{max} = +\infty \rightarrow \frac{2}{1} = 2$

$x^2 - 1 = 0 \rightarrow R_f = (-\infty, -2] \cup (2, +\infty)$

الف) $y = \frac{2 \sin x - 1}{\sin x + 2}$

$y \sin x + 2y = 2 \sin x - 1 \rightarrow (y-2) \sin x = -(2y+1)$

$\sin x = \frac{2y+1}{2-y}$

$-1 \leq \sin x \leq 1 \rightarrow -1 \leq \frac{2y+1}{2-y} \leq 1$

$\frac{2y+1}{2-y} \leq 1$

$\frac{1}{1+y} \leq \frac{2y}{1-y} \rightarrow \frac{1}{1+y} \leq 2y$ (II)

ب) $y = \frac{\sin x - 1}{\sin x + 2}$

$\sin x_{max} = 1$

$\frac{1-1}{1+2} = -\frac{1}{3}$

$\sin x_{min} = -1 \rightarrow \frac{-1-1}{-1+2} = -2$

$R_f = [-\frac{1}{3}, \frac{1}{3}]$

$$f(x) = \frac{x^2 - x + \frac{1}{2}}{x^2 - x + 1} \rightarrow x^2 - x + 1 \neq 0 \rightarrow \Delta < 0 \quad D_f = \mathbb{R}$$

$$y = \frac{x^2 - x + \frac{1}{2}}{x^2 - x + 1} \rightarrow yx^2 - yx + y = x^2 - x + \frac{1}{2} \rightarrow 0 = (1-y)x^2 + (y-1)x + \frac{1}{2}-y$$

$$x = \frac{y^2 - y + 1}{(1-y) \pm \sqrt{(y-1)^2 - (1-y)(1-y)}} \quad ; y \neq 1$$

$$0 = \frac{1}{2} - y \rightarrow y = \frac{1}{2} \text{ } \delta\delta\delta \quad ; y = 1$$

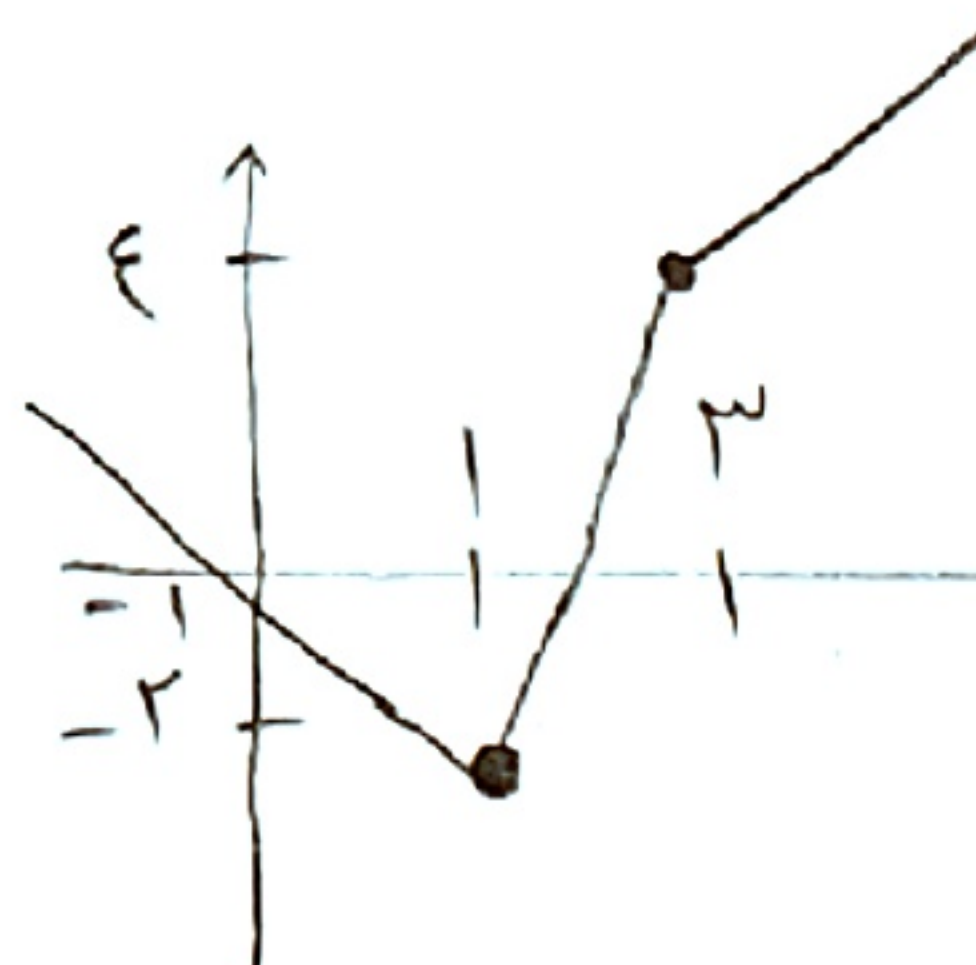
$$y^2 + 1 - 2y - (1-y)(1-y) \geq 0$$

$$-2(y^2 - y) \geq 0 \rightarrow y - y^2 \geq 0$$

$$R_f = [0, +1]$$

$$a) y = |2x-2| - |x-3|$$

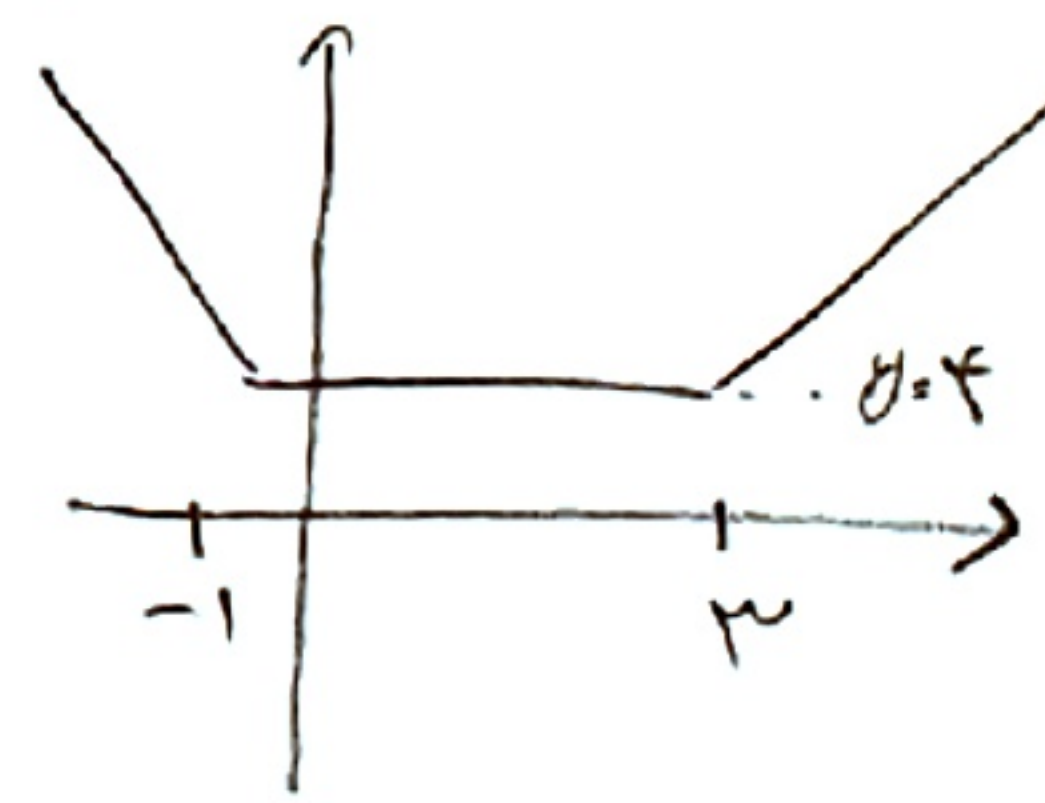
$$\begin{cases} -x-1 & x < 1 \\ 3x-8 & 1 \leq x \leq 3 \\ x+1 & x > 3 \end{cases}$$



$$R_f = [-2, +\infty)$$

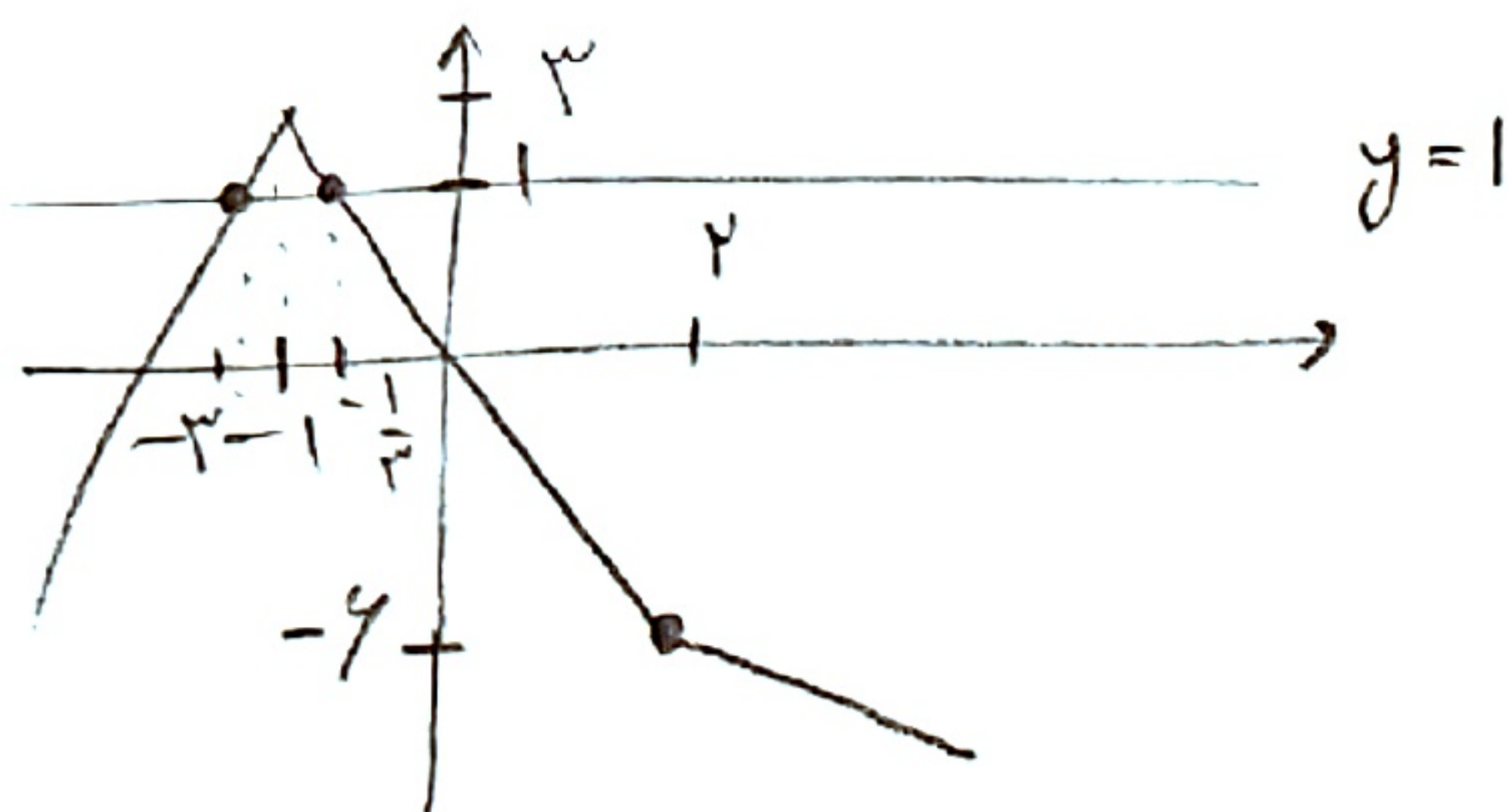
$$b) y = |x-2| + |x+1|$$

$$\begin{cases} -2x+3 & x < -1 \\ x+3 & -1 \leq x \leq 2 \\ 2x-1 & x > 2 \end{cases}$$



$$R_f = [2, +\infty)$$

$$y = |x-2| - |2x+2| \leq 1$$



$$\begin{cases} x+3 & x < -1 \\ -3x & -1 \leq x \leq 2 \\ -x-4 & x > 2 \end{cases} \rightarrow x \in (-\infty, -1] \cup [-\frac{1}{3}, +\infty)$$

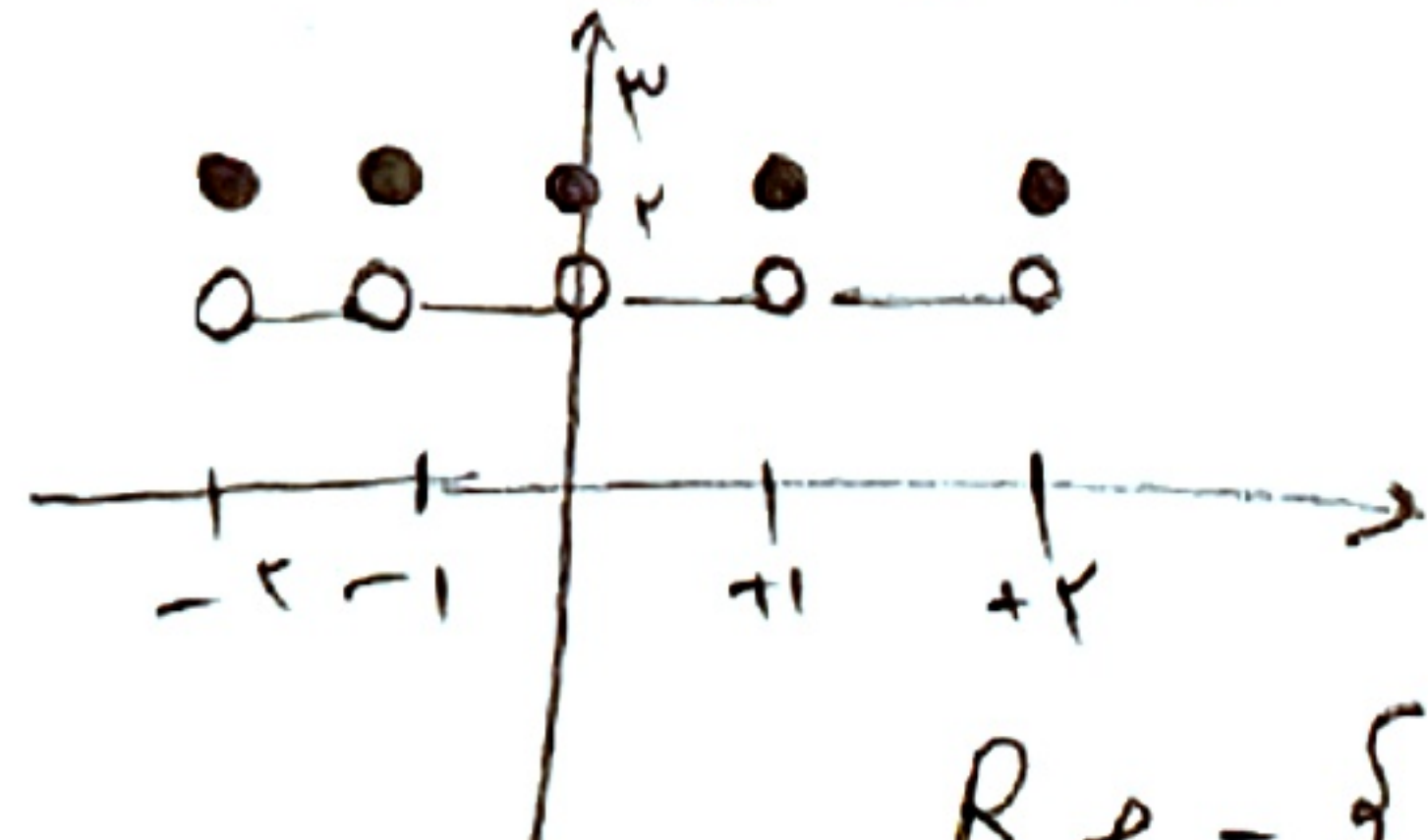
$$\begin{cases} x+3 & x < -1 \\ -3x & -1 \leq x \leq 2 \\ -x-4 & x > 2 \end{cases}$$

$$-3x = 1 \rightarrow x = -\frac{1}{3}$$

$$x+3 = 1 \rightarrow x = -2$$

$$a) y = [x] + [3-x]$$

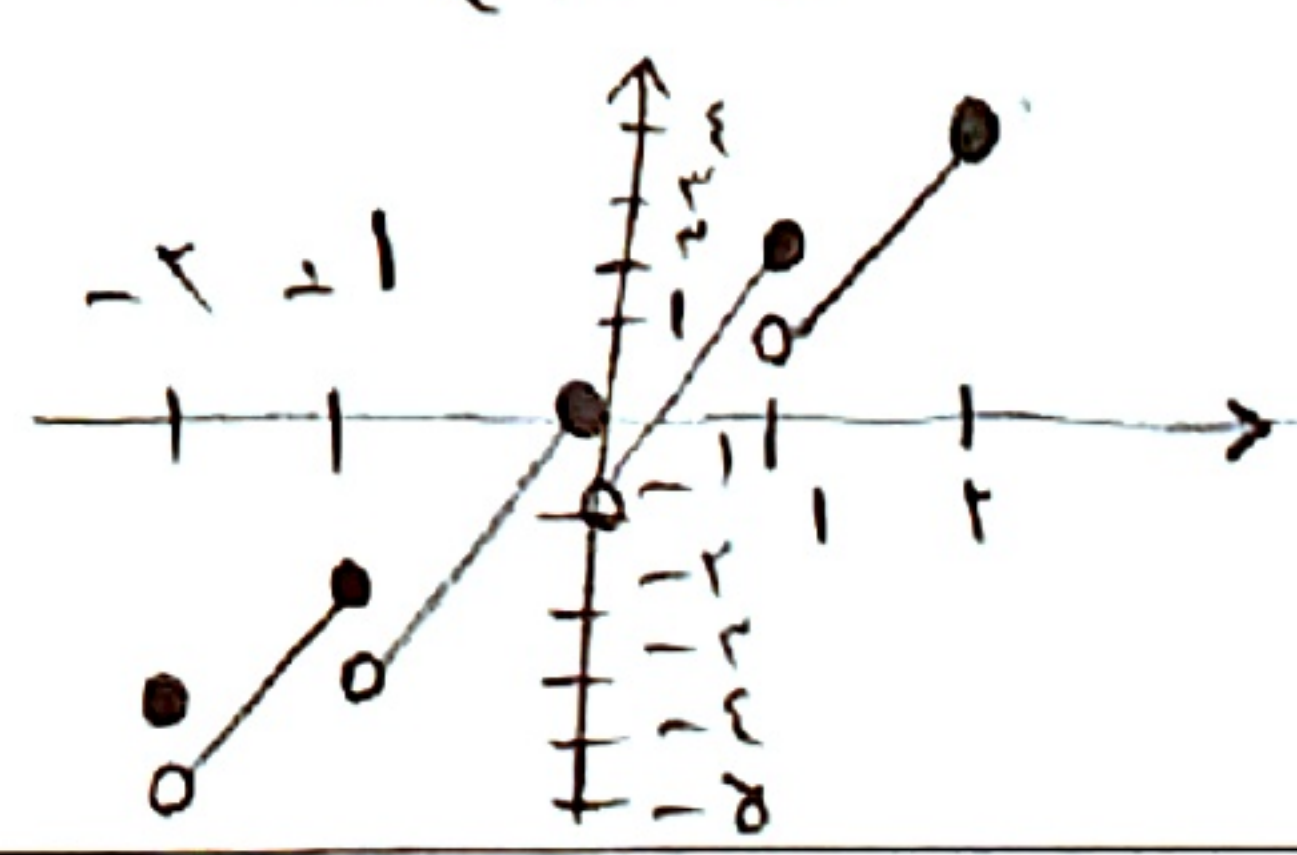
$$= [x] + [-x] + 3$$



$$R_f = \{3\}$$

$$b) y = 3x - [x]$$

$$\begin{aligned} f(-2) &= -2 & f(-1) &= -1 & f(0) &= 0 \\ f(1) &= 2 & f(2) &= 4 \end{aligned}$$



$$R_f = [-1, 3)$$

$$R_f = (-1, 3]$$

$$a) y = \sin^2 x - \sin x + 1$$

$$\sin x_{\min} = -1 \rightarrow y_{\max} = +3$$

$$\sin x_{\max} = 1 \rightarrow y = +1$$

$$-\frac{b}{2a} = \frac{1}{2} \rightarrow y_{\min} = \frac{3}{2}$$

$$R_f = [\frac{3}{2}, +3]$$

$$b) y = \sin^2 x + \sin^2 x + 1$$

$$\sin^2 x_{\min} = 0 \rightarrow y_{\min} = +1$$

$$\sin^2 x_{\max} = 1 \rightarrow y_{\max} = +3$$

$$-\frac{b}{2a} = -\frac{1}{2} \text{ } \delta\delta\delta$$

$$R_f = [1, +3]$$