

$$y = \frac{x^2 + x + 2}{x - 1} \rightarrow yx - y = x^2 + x + 2 \rightarrow 1 = x^2 + (1 - y)x + 2 + y$$

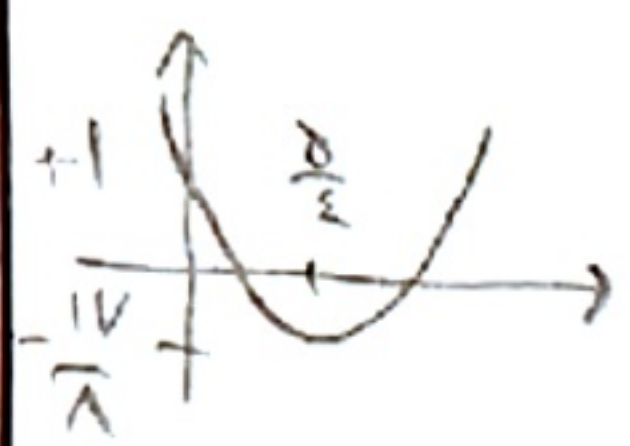
$$x = \frac{-(1 - y) \pm \sqrt{(1 - y)^2 - 4(2 + y)}}{2} \rightarrow (1 - y)^2 - 4(y + 2) \geq 0 \rightarrow y^2 - 6y - 7 \geq 0$$

$$y \notin (-1, +\infty)$$

$$R_f = \mathbb{R} - (-1, +\infty)$$

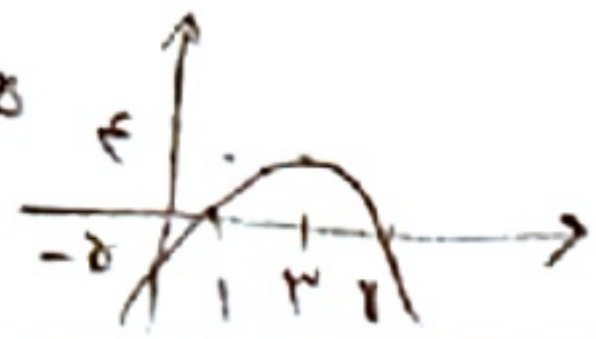
ا) $y = 2x^2 - 8x + 1$

$R_f = [-\frac{17}{2}, +\infty)$



ب) $y = \sqrt{-x^2 + 4x - 8}$ $\rightarrow (-\infty, 4]$

$-x^2 + 4x - 8 \geq 0 \rightarrow R = (-\infty, 4] \quad R_f = [0, 2]$



ج) $y = \sqrt{x^2 - 3x^2 + 4x + 1}$

$g(x) = x^2 - 3x^2 + 4x + 1 \rightarrow R_g = \mathbb{R}$

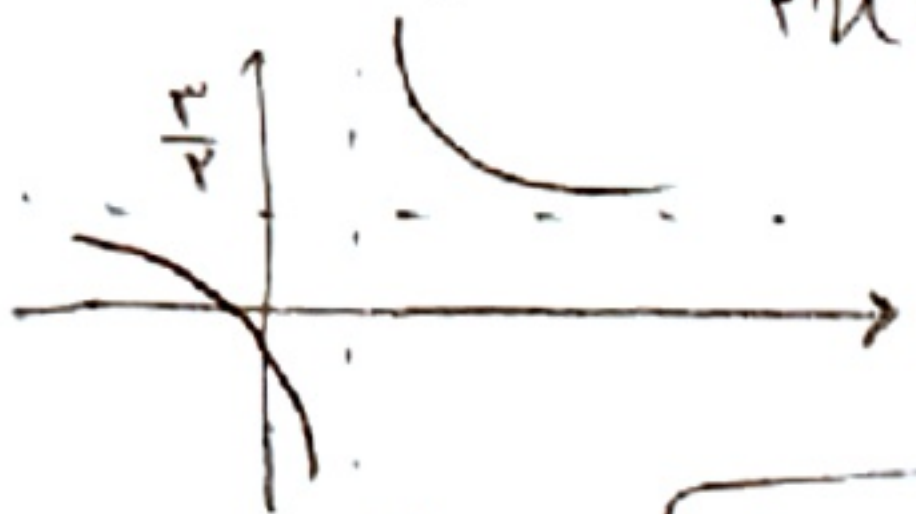
$y = \sqrt{(-\infty, +\infty)} \rightarrow R_f = [0, +\infty)$

د) $\log(x^2 - 3x^2 + 4x + 1)$

$g(x) = x^2 - 3x^2 + 4x + 1 \rightarrow R_g = (-\infty, +\infty)$

$(-\infty, +\infty) \xrightarrow{\log} R_f = (-\infty, +\infty)$

ا) $y = \frac{3x + 1}{x - 2} \quad R_f = \mathbb{R} - \{\frac{1}{2}\}$



ب) $y = \sqrt{\frac{3x + 1}{x - 2}} \rightarrow \sqrt{(-\infty, +\infty) - \{\frac{1}{2}\}} \rightarrow R_f = [0, +\infty) - \{\frac{1}{2}\}$

$\frac{3x + 1}{x - 2} \rightarrow R = \mathbb{R} - \{\frac{1}{2}\}$

ج) $y = \frac{x^2 + 1}{\sqrt{x^2 + 3}} = \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}}$

$y_{min} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \rightarrow R_f = [\frac{\sqrt{3}}{3}, +\infty)$

$\frac{x^2 + 1}{\sqrt{x^2 + 3}} \geq \sqrt{x^2 + 3} \rightarrow x_{min} = 0 \rightarrow y_{min} = \frac{\sqrt{3}}{3}$

د) $y = x + \frac{1}{x^2 + 1} \rightarrow \frac{x^2}{x^2 + 1} > 0$

$x_{min} = 0 \rightarrow y_{min} = \frac{1}{2} \rightarrow R_f = [\frac{1}{2}, +\infty)$

1) $y = \frac{3x^2 + 1}{x^2 - 1} \rightarrow$

$y \neq 1; x = \frac{0 \pm \sqrt{1(y - 3)(y + 1)}}{3 - y}$

$y = 3; 0 = 1 + y \rightarrow y = -1 \rightarrow 0 \in \mathbb{R}$

$yx^2 - y = 3x^2 + 1 \rightarrow 0 = (3 - y)x^2 + 1 + y$

$1(y + 1)(y - 3) \geq 0 \rightarrow y \in \mathbb{R} - (-1, +3]$

$R_f = (-\infty, -1] \cup (3, +\infty)$

2) $\frac{3x^2 + 1}{x^2 - 1} \rightarrow x^2_{min} = 0 \rightarrow \frac{1}{-1} = -1$

$x^2_{max} = +\infty \rightarrow \frac{3}{1} = 3$

$x^2 - 1 = 0 \rightarrow R_f = (-\infty, -1] \cup (3, +\infty)$

1) $y = \frac{3 \sin x - 1}{\sin x + 2} \rightarrow$

$\sin x = \frac{3y + 1}{2 - y}$

$-1 \leq \sin x \leq 1 \rightarrow -1 \leq \frac{3y + 1}{2 - y} \leq 1$

$(I) \cap (II) = R_f = [-\frac{3}{2}, \frac{1}{2}]$

$\frac{3y - 1}{2 - y} \leq 1$

$\frac{1}{1 + 1} \leq \frac{3y}{2 - y} \rightarrow \frac{1}{2} \leq y < 2$ (II)

$\frac{3y + 1}{2 - y} \geq -1 \rightarrow \frac{3}{2} \leq y < 2$ (I)

2) $y = \frac{\sin x - 1}{\sin x + 2} \rightarrow$

$\sin x_{max} = 1 \rightarrow \frac{1 - 1}{1 + 2} = -\frac{1}{3}$

$\sin x_{min} = -1 \rightarrow \frac{-1 - 1}{-1 + 2} = -2$

$R_f = [-2, -\frac{1}{3}]$

$$f(x) = \frac{x^2 - x + \frac{1}{\varepsilon}}{x^2 - x + 1} \rightarrow x^2 - x + 1 \neq 0 \rightarrow \Delta < 0 \quad D_f = \mathbb{R}$$

$$y = \frac{x^2 - x + \frac{1}{\varepsilon}}{x^2 - x + 1} \rightarrow yx^2 - yx + y = x^2 - x + \frac{1}{\varepsilon} \rightarrow 0 = (1-y)x^2 + (y-1)x + \frac{1}{\varepsilon} - y$$

$$x = \frac{y^2 - y + 1}{(1-y) \pm \sqrt{(y-1)^2 - (1-\varepsilon y)(1-y)}} \quad ; y \neq 1$$

$$0 = \frac{1}{\varepsilon} - y \rightarrow y = \frac{1}{\varepsilon} \text{ o.s. } ; y = 1$$

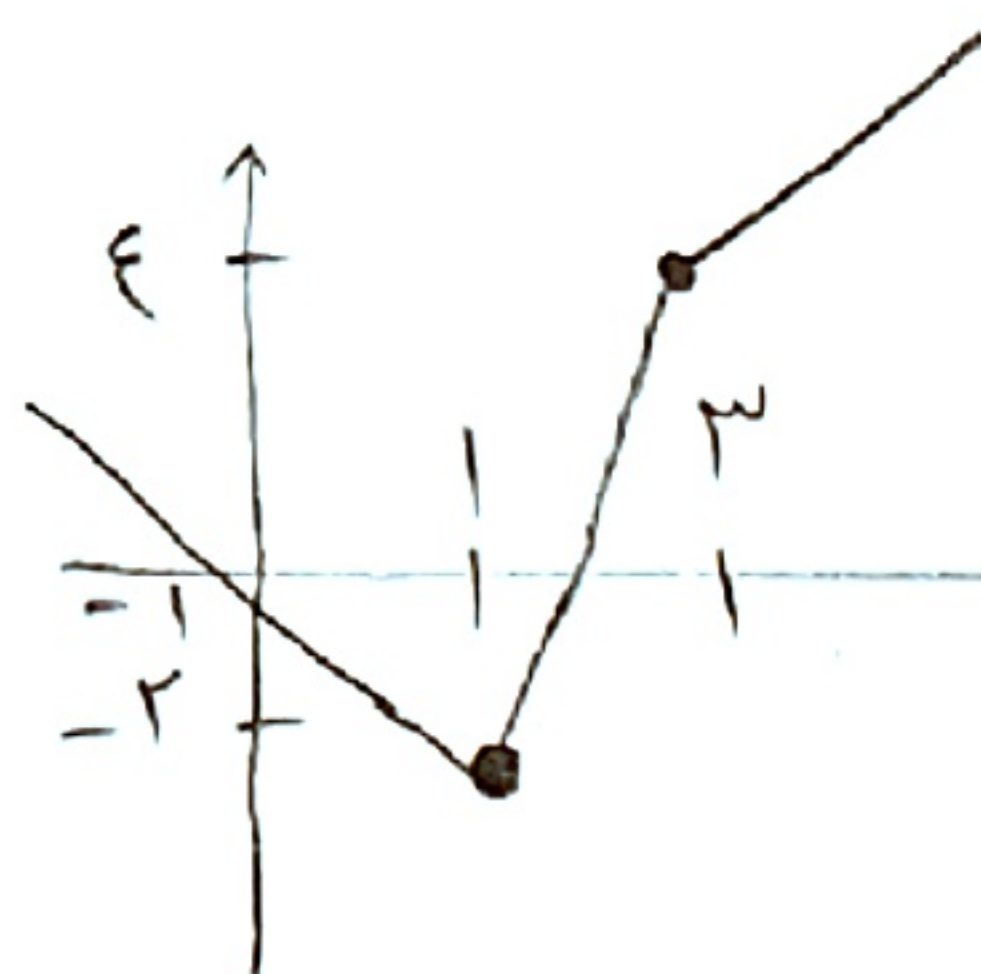
$$y^2 + 1 - 2y - (\varepsilon y^2 - 2y + 1) \geq 0$$

$$- \varepsilon (y^2 - y) \geq 0 \rightarrow y - y^2 \geq 0$$

$$R_f = [0, +1)$$

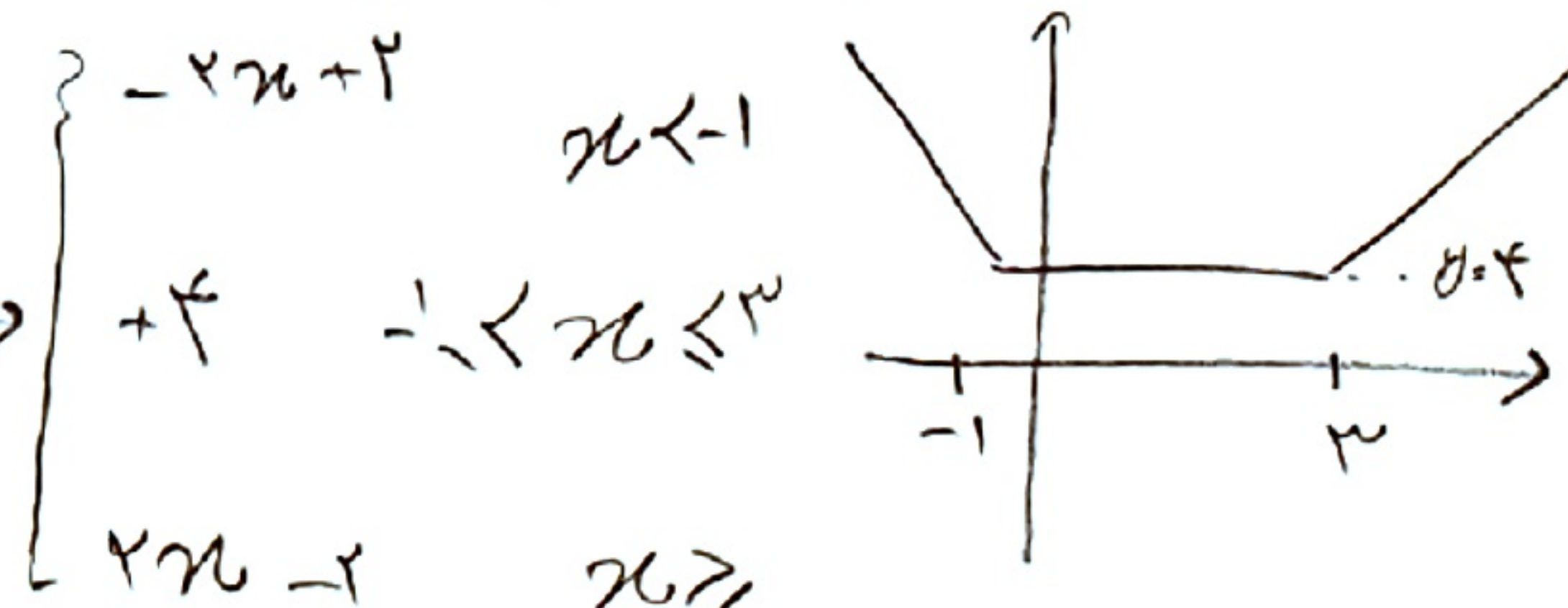
$$a) y = |2x-2| - |x-3|$$

$$\begin{cases} -x-1 & x < 1 \\ 3x-8 & 1 \leq x \leq 3 \\ x+1 & x > 3 \end{cases}$$



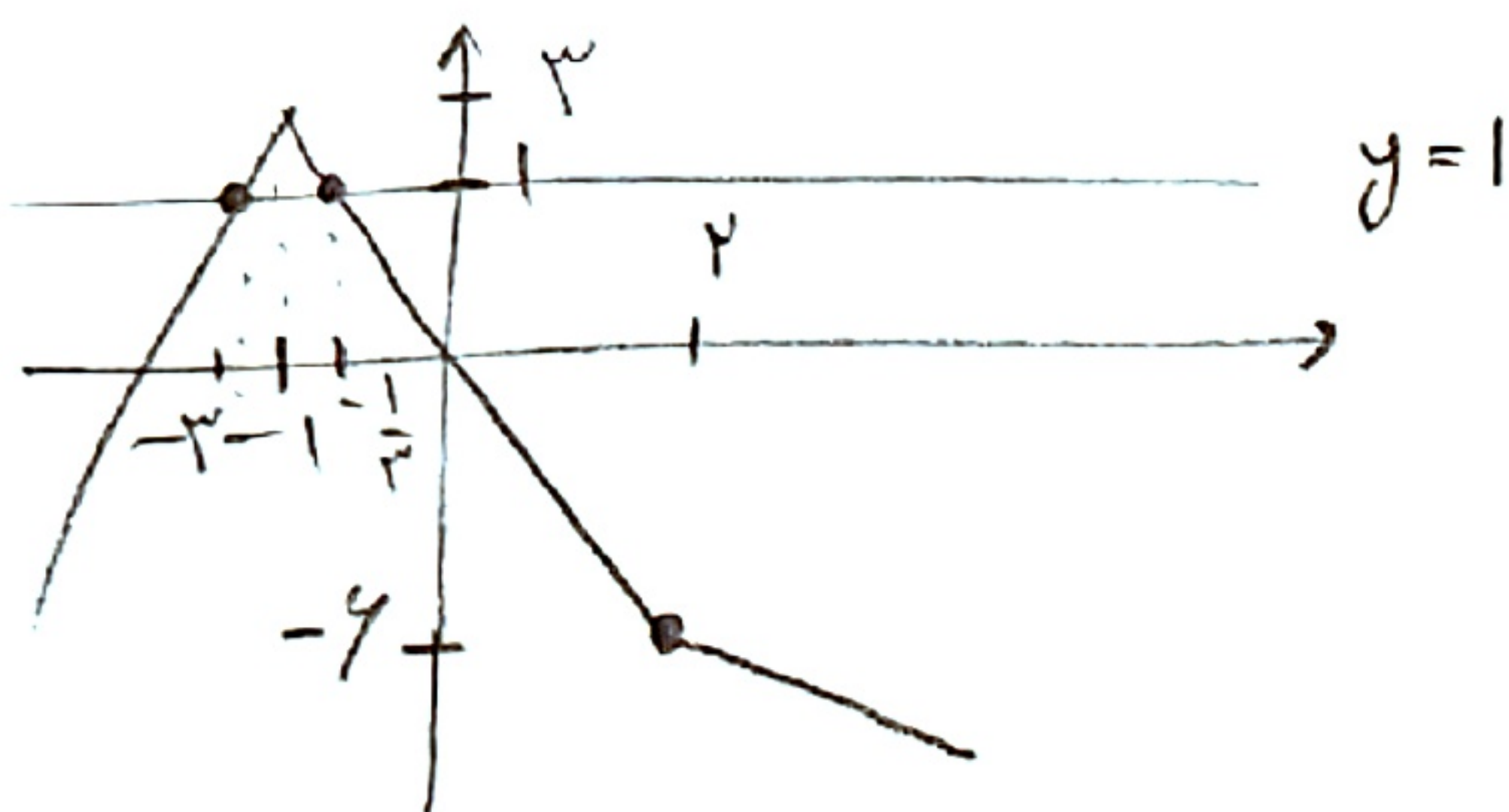
$$R_f = [-2, +\infty)$$

$$b) y = |x-2| + |x+1|$$



$$R_f = [1, +\infty)$$

$$y = |x-2| - |2x+2| \leq 1$$



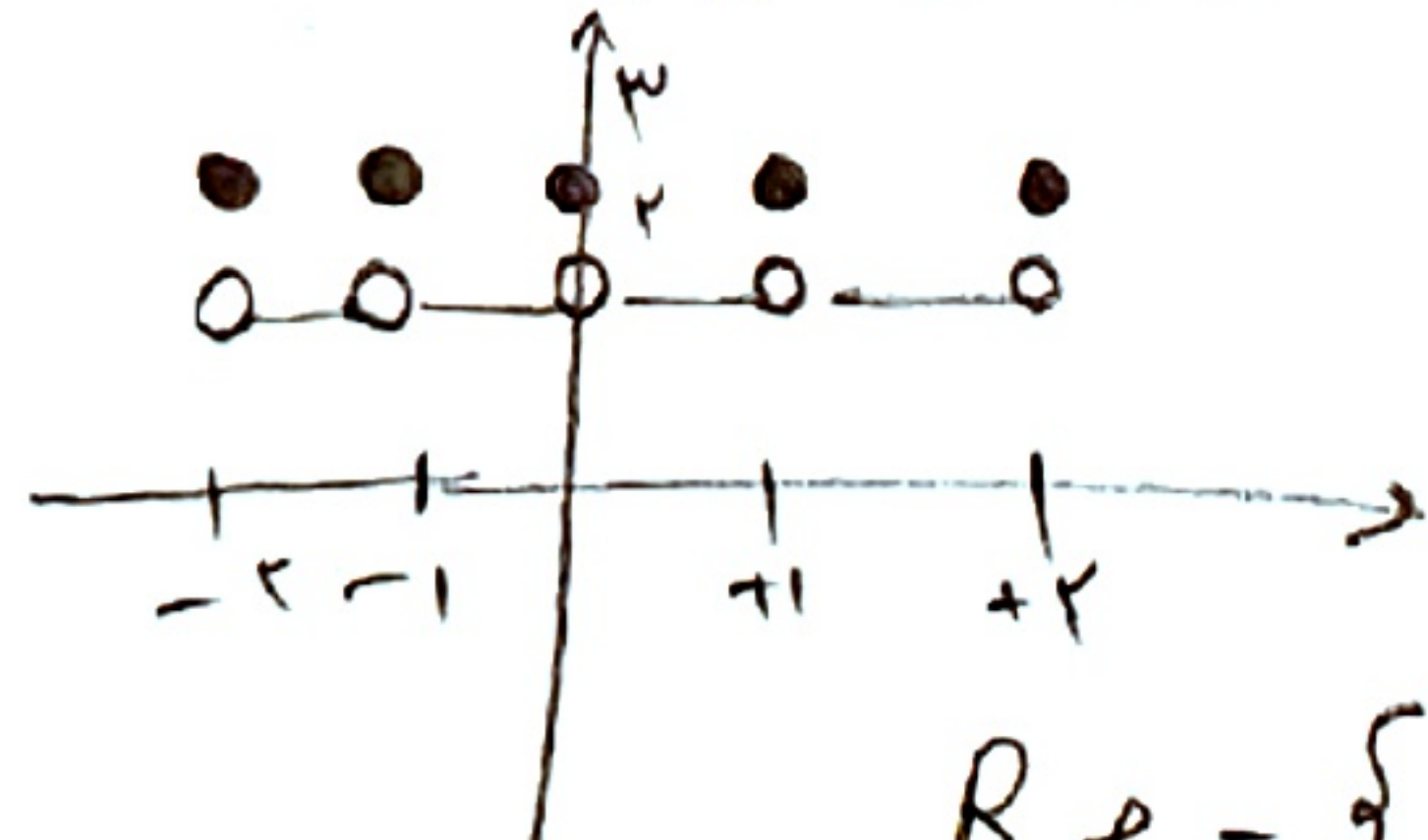
$$\begin{cases} x+1 & x < -1 \\ -2x & -1 \leq x \leq 2 \\ -x-4 & x > 2 \end{cases} \rightarrow x \in (-\infty, -1] \cup [-\frac{1}{2}, +\infty)$$

$$-2x = 1 \rightarrow x = -\frac{1}{2}$$

$$x+1 = 1 \rightarrow x = 0$$

$$a) y = [x] + [3-x]$$

$$= [x] + [-x] + 3$$

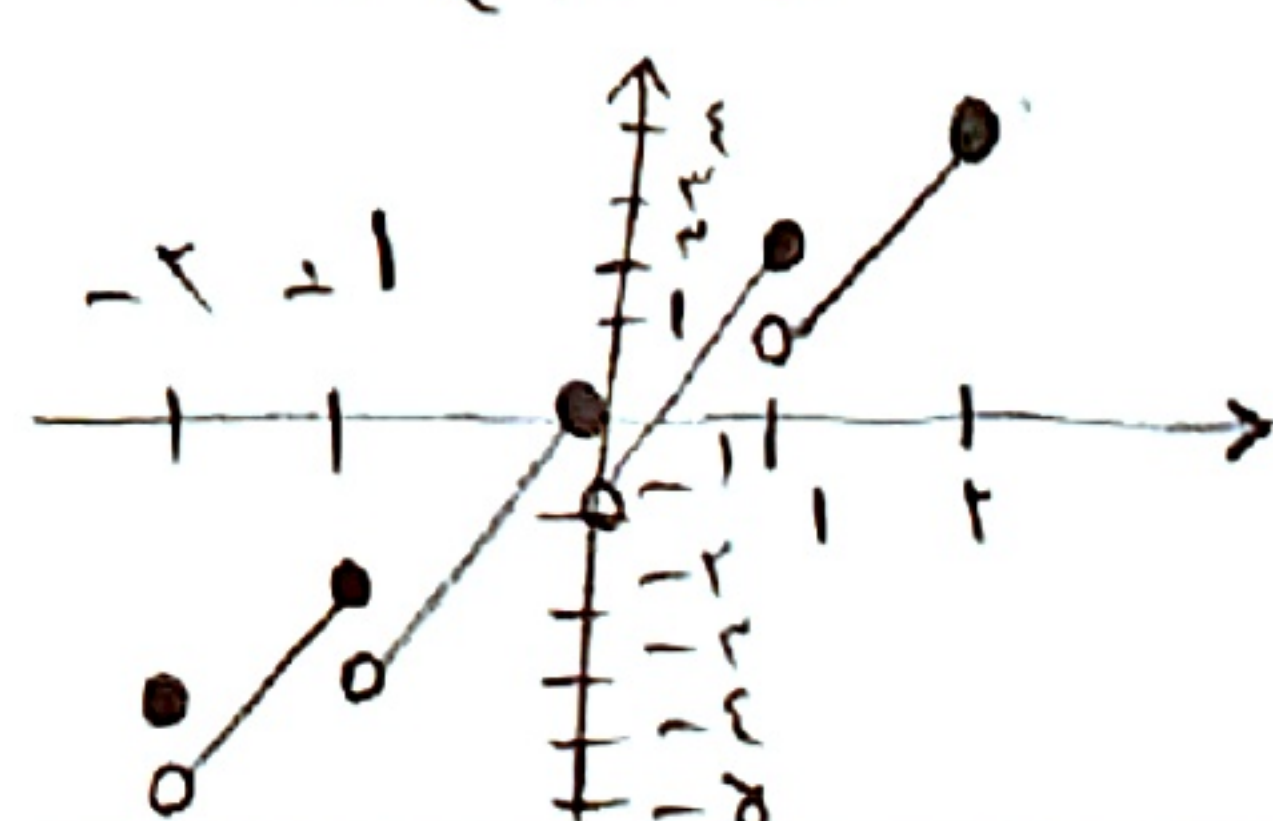


$$R_f = \{3, 4\}$$

$$b) y = 3x - [x]$$

$$f(-2) = -2 \quad f(-1) = -1 \quad f(0) = 0$$

$$f(1) = 2 \quad f(2) = 4$$



$$R_f = (-\infty, +\infty]$$

$$a) y = \sin^2 x - \sin x + 1$$

$$\sin x_{\min} = -1 \rightarrow y_{\max} = +3$$

$$\sin x_{\max} = 1 \rightarrow y = +1$$

$$-\frac{b}{4a} = \frac{1}{4} \rightarrow y_{\min} = \frac{3}{4}$$

$$R_f = [\frac{3}{4}, +3]$$

$$b) y = \sin^2 x + \sin^2 x + 1$$

$$\sin^2 x_{\min} = 0 \rightarrow y_{\min} = +1$$

$$\sin^2 x_{\max} = 1 \rightarrow y_{\max} = +3$$

$$-\frac{b}{4a} = -\frac{1}{4} \text{ o.s. }$$

$$R_f = [1, +3]$$