

الف) $y = (x - 2[\frac{x}{2}])^2 \xrightarrow{x=0} \Rightarrow 0 - 2(0) \Rightarrow y_{\min}$
 $\Rightarrow R_F = [0, +\infty)$

ب) $\log y = \sin x + 4 \xrightarrow{-1 \leq \sin x \leq 1} \Rightarrow 3 \leq \sin x + 4 \leq 5 \Rightarrow 3 \leq \log y \leq 5$
 $10^3 \leq y \leq 10^5$

$F(x) = \frac{x^2-4}{x^2-9} \rightarrow x^2=0 \Rightarrow y_1 = \frac{-4}{-9} = \frac{4}{9}$ و $y_2 = \text{نسبت ضمیمه} = 1$

$\xrightarrow{\text{خرج صفر شود}} \Rightarrow R'_F = (-\infty, \frac{4}{9}] \cup (1, +\infty) \Rightarrow R_F = \sqrt{R'_F}$ $\xrightarrow{\text{جذر را میگیریم}}$
 $\Rightarrow R_F = [0, \frac{2}{3}] \cup (1, +\infty) \Rightarrow a+b+c = 0 + \frac{2}{3} + 1 = \frac{5}{3}$

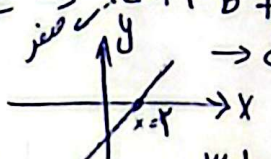
الف) $x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} = (\sqrt{x + \frac{1}{x}} + 1)^2 - 1 \xrightarrow{x \geq 0} (\sqrt{2} + 1)^2 - 1$
 $\xrightarrow{x + \frac{1}{x} \geq 2} \Rightarrow y_{\min} = 2 + 1 + 2\sqrt{2} - 1 = 2 + 2\sqrt{2} \Rightarrow R_F = [2 + 2\sqrt{2}, +\infty)$

ب) $-\sin^2 x + 2\sin x + 3 \left\{ \begin{array}{l} 1 \rightarrow -(-1)^2 + 2(1) + 3 = 4 \\ -1 \rightarrow -(-1)^2 + 2(-1) + 3 = 0 \\ -\frac{b}{2a} = 1 \end{array} \right\} \Rightarrow R_F = [0, 4]$

$D_F = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \Rightarrow f(x) = (\frac{-1}{3}(1) + 10)(\frac{-1}{3}(2) + 10) + \dots + (\frac{-1}{3}(9) + 10)$

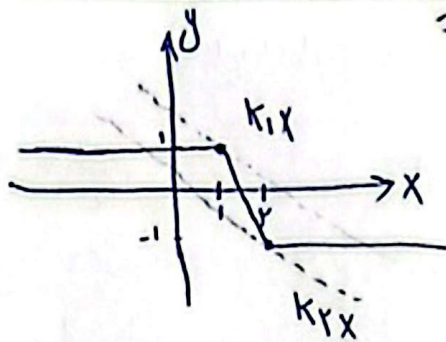
$\xrightarrow{\text{مقدارگیری}} \frac{-1}{3}(1+2+\dots+9) + 9 \times 10 = \frac{-1}{3}(\frac{9 \times 10}{2}) + 90 = 75$
 $\xrightarrow{\text{جمع بدو}}$

$f(3) = 1 \Rightarrow 1 = \sqrt{9a + 3b + c} \Rightarrow 9a + 3b + c = 1$

$ax^2 + bx + c \geq 0 \xrightarrow{D_F \geq 2} \xrightarrow{\text{منحنی}}$  $\rightarrow a=0 \rightarrow bx+c$

$x=2 \rightarrow 2b+c=0 \Rightarrow c=-2b$, $3b+c=1 \Rightarrow b=1, c=-2$

$g(x) = \sqrt{x^2+4} \rightarrow x^2=0 \Rightarrow y_{\min} = \sqrt{4} = 2 \Rightarrow R_F = [2, +\infty)$



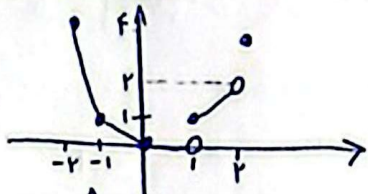
$$f(x) = |x-2| - |x-1|$$

$$k_1 x \rightarrow x=1 \Rightarrow k_1 \cdot 1 = 1 \Rightarrow k_1 = 1$$

$$k_2 x \rightarrow x=2 \Rightarrow 2k_2 = -1 \Rightarrow k_2 = -\frac{1}{2}$$

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الف



$$(-2, -1] \rightarrow -2x$$

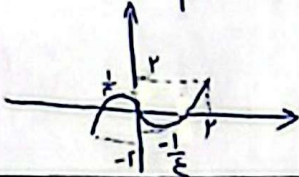
$$(-1, 0] \rightarrow -x$$

$$(0, 1] \rightarrow 0$$

$$(1, 2] \rightarrow x$$

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ب)ا



$$y \begin{cases} x \geq 0: & x^2 - x \rightarrow \frac{-b}{2a} = \frac{1}{2} \rightarrow \frac{1}{2} \\ x < 0: & -x^2 - x \rightarrow \frac{-b}{2a} = \frac{1}{2} \rightarrow \frac{1}{2} \end{cases}$$

$$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} = \frac{x+1+x+1}{x^2+x} = \frac{2x+2}{x(x+1)} = \frac{2}{x}$$

مخرج صفر $\rightarrow x=0$
 $x+1=0 \Rightarrow x=-1$ } $Df = R - \{0, -1\}$

$u = \begin{cases} 0 \rightarrow \frac{2}{0} \rightarrow \infty \\ -1 \rightarrow \frac{2}{-1} \rightarrow -2 \end{cases}$ } $\Rightarrow Rf = R - \{0, -2\}$

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$$f(x) = \frac{x - \sqrt{x} + 3}{x - 2\sqrt{x} + 1} = \frac{(\sqrt{x}-1)^2 - 1}{(\sqrt{x}-1)^2} \xrightarrow{(\sqrt{x}-1) \rightarrow t} \frac{(t-1)^2 - 1}{t^2}$$

$$\Rightarrow \frac{t^2 - 2t + 1 - 1}{t^2} = \frac{t^2 - 2t}{t^2} \Rightarrow y = \frac{x-2}{x} \Rightarrow yx = x-2$$

$$Rf = R - \{1\} \quad \Leftarrow y \neq 1 \quad \Leftarrow x = \frac{-2}{y-1}$$

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$$\frac{x^3 + 2x^2 - x - 2}{x^3 + x} \Big| \frac{x^2}{x+2} \Rightarrow f(x) = x+2$$

مخرج صفر $\rightarrow x^2 - 1 = (x-1)(x+1) \Rightarrow Df = R - \{1, -1\}$

$f(x) \rightarrow \begin{cases} x=1 & x+2=3 \\ x=-1 & x+2=1 \end{cases} \Rightarrow Rf = R - \{1, 3\}$

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