آزمون نوبت اول

۱) الف) $\left(۲\left(\frac{x}{۲}-\left[\frac{x}{۲}\right]\right)\right)^۲$ $\quad ۲\left(\frac{x}{۲}-\left[\frac{x}{۲}\right]\right)\to[۰,۲)$ $\quad \left(۲\left(\frac{x}{۲}-\left[\frac{x}{۲}\right]\right)\right)^۲$

$[۰,۱)$

$R_f=[۰,۴)$ ✓ ۱۹,۵

ب) $\log y=\sin x+۴$ $\quad -۱\le\sin x\le ۱$ $\quad ۳\le\sin x+۴\le ۵$

$۳\le\log y\le ۵$ $\quad ۱۰^۳\le y\le ۱۰^۵$ $\quad R_f=[۱۰^۳,۱۰^۵]$ ✓ (۴)

۲- $\frac{a\square+b}{c\square+d}$ $\quad \square=x^۲\to x^۲\begin{cases}\min\to ۰\\ \max\to+\infty\end{cases}$ $\quad \frac{x^۲-۴}{x^۲-۹}\to\begin{cases}x^۲=۰\to ۴/۹\\ x^۲=+\infty\to ۱\end{cases}$ مخرج کسر نباید صفر شود

$(-\infty,\frac{۴}{۹}]\cup(۱,+\infty)\to R_f=[۰,\frac{۲}{۳}]\cup(۱,+\infty)$ ✓ $\quad c=۱$ $\quad b=\frac{۲}{۳}$ $\quad a=۰$

$a+b+c=\frac{۵}{۳}$ ✓ (۲)

۳- الف) $f(x)=x+\frac{۱}{x}+۲\sqrt{x+\frac{۱}{x}}\to f(x)=\underbrace{\left(\sqrt{x+\frac{۱}{x}}+۱\right)^۲}_{(۱,+\infty)}-۱$ ✓ $\Rightarrow R_f=(۰,+\infty)$

$\sqrt{x+\frac{۱}{x}}\xrightarrow{\min}\sqrt{۲}$ $\quad R_f=[۲+۲\sqrt{۲},+\infty)$

ب) $y=-(\sin x-۳)(\sin x+۱)\to y=-\sin^۲x+۲\sin x+۳\to y=a\square^۲+b\square+c$ (۱,۵)

$\square=\sin x$ $\quad \sin x\begin{cases}\min\to -۱\to -۱-۲+۳=۰\\ \max\to +۱\to -۱+۲+۳=۴\\ -b/۲a\to\frac{-۲}{-۲}=۱\to ۴\end{cases}$ $\quad R_f=[۰,۴]$ ✓

۴- $f(x)=-\frac{۱}{۳}x+۱۰$ $\quad D_f=\{۱,۲,۳,۴,\dots,۹\}\to R_f=\{۲۹/۳,۲۸/۳,۲۷/۳,\dots,۲۱/۳\}$

$x=۱\to y=۲۹/۳$

$x=۲\to y=۲۸/۳$

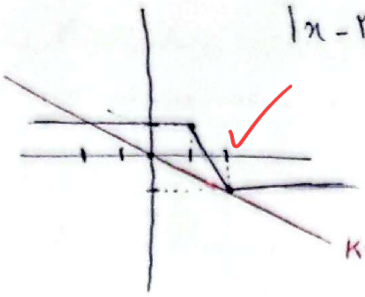
$\vdots$

$۲۱/۳+۲۲/۳+۲۳/۳+\dots+۲۹/۳=۹\times\frac{۲۹/۳+۲۱/۳}{۲}=۹\times\frac{۲۵}{۳}=۷۵$ ✓ (۲)

۵- $D_f\to\frac{۲}{-۱+}$ $\quad a=۰$ $\quad f(x)=\sqrt{bx+c}\to\begin{cases}۲b+c=۰\\ ۳b+c=۱\end{cases}\to b=۱$ $\quad c=-۲$ ✓ $\quad a=۰$ ✓

$g(x)=\sqrt{x^۲+۴}\to x^۲+۴\to[۴,+\infty)\Rightarrow\sqrt{[۴,+\infty)}\Rightarrow R_f=[۲,+\infty)$ ✓ (۴)

$|x-2|-|x-1| = -\frac{1}{2}x$

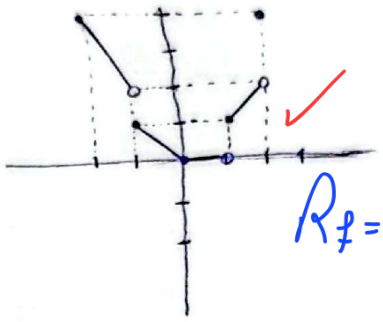


$Kx \longrightarrow (2,-1)$ $\quad 2K=-1$ $\quad K=-\frac{1}{2}$ ✓ (2)

-۶

-۷

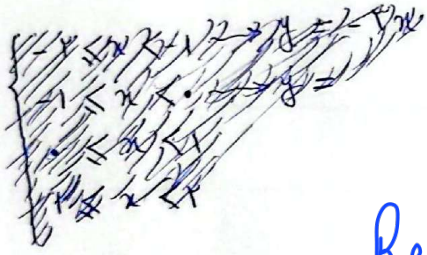
الف)
$$\begin{cases} -2 \le x < -1 \longrightarrow y = -2x \\ -1 \le x < 0 \longrightarrow y = -x \\ 0 \le x < 1 \longrightarrow y = 0 \\ 1 \le x < 2 \longrightarrow y = x \end{cases}$$ ✓



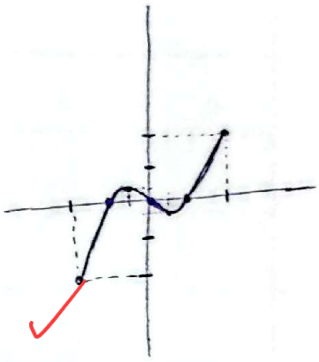
(1,5)

$R_f = [0,4] - \{2\}$

ب)



$$\begin{array}{r|l} -x^2-x & x^2-x \\ \hline \end{array}$$ ✓



$R_f = [-2,2]$

-۸

$$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} \longrightarrow \frac{x+1+x+1}{x^2+x} = \frac{2x+2}{x^2+x} = \frac{2(x+1)}{x(x+1)} = \frac{2}{x}$$ ✓

$D_f = R - \{0,-1\}$ $\quad x=-1 \longrightarrow y=-2$

$R_f = R - \{0,-2\} \longrightarrow a+b=-2$ ✓ (2)

-۹

$$f(x) = \frac{x-4\sqrt{x}+3}{x-2\sqrt{x}+1} \longrightarrow \frac{(\sqrt{x}-3)(\sqrt{x}-1)}{(\sqrt{x}-1)^2} = \frac{\sqrt{x}-3}{\sqrt{x}-1} \longrightarrow \frac{a\square+b}{c\square+d}$$ ✓ (2)

$\square = \sqrt{x}$ $\quad$ min $\to 0 \to 3$ $\quad$ max $\to +\infty \to 1$

$(-\infty,1) \cup [3,+\infty)$ ✓

$$\begin{array}{r|l} x^3+2x^2-x-2 & x-1 \\ \hline -x^3+x^2 & x^2+3x+2 \\ \hline 3x^2-x-2 & \\ -3x^2+3x & \\ \hline 2x-2 & \\ 2x-2 & \\ \hline 0 & \end{array}$$

$$\frac{(x-1)(x+1)(x+2)}{(x-1)(x+1)} = x+2$$ ✓

-۱۰ $D_f = R - \{\pm 1\}$ (2)

$x=1 \longrightarrow y=3$

$x=-1 \longrightarrow y=1$

$R_f = R - \{1,3\}$ ✓ $\quad 3+1=4$ ✓

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