

Subject:

Date:

① الف) $y = (x - [\frac{x}{2}])^2 = (x - (\frac{x}{2} - [\frac{x}{2}]))^2 = (\frac{x}{2} - [\frac{x}{2}])^2 \rightarrow \frac{x}{2} - [\frac{x}{2}] \in [0, \frac{1}{2}]$
 $y \in [0, \frac{1}{4}]$
 ب) $-\sin x + \log y - 1 \approx \log y = \sin x + 1$
 $y = [10^{\sin x + 1}]$

② $f(x) = \sqrt{\frac{x^2 - 4}{x^2 - 9}}$
 $\lim_{x \rightarrow \infty} \frac{x^2 - 4}{x^2 - 9} = 1 \rightarrow \sqrt{1} = 1$
 $\lim_{x \rightarrow 3} \frac{x^2 - 4}{x^2 - 9} = \frac{0}{0} = \frac{4}{9}$
 $\lim_{x \rightarrow -3} \frac{x^2 - 4}{x^2 - 9} = \frac{0}{0} = \frac{4}{9}$
 $D_f = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

③ الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x+1}$
 $x > 0$
 $\lim_{x \rightarrow 0} f(x) = \infty$
 $\lim_{x \rightarrow \infty} f(x) = \infty$
 $f'(x) = 1 - \frac{1}{x^2} + \frac{1}{\sqrt{x+1}} = 0$
 $x = 1$
 $f(1) = 1 + 1 + 2\sqrt{2} = 2 + 2\sqrt{2}$
 $D_f = [1, \infty)$

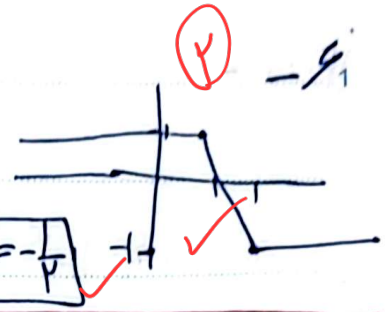
ب) $y = (3 - \sin x)(1 + \sin x) = 3 + 3\sin x - \sin^2 x = -\sin^2 x + 3\sin x + 3$
 $y \in [2, 4]$

④ $f(x) = -\frac{x}{3} + 10$
 $f(1) = -\frac{1}{3} + 10$
 $f(9) = -\frac{9}{3} + 10 = 7$
 $D_f = \{1, 2, \dots, 9\}$

⑤ $f(x) = \sqrt{ax^2 + bx + c}$
 $x = 2 \Rightarrow y = 0 \Rightarrow 4a + 2b + c = 0$
 $f(3) = 1 \Rightarrow 9a + 3b + c = 1$
 $a = 0, b = 1, c = -2$
 $g(x) = \sqrt{bx^2 + ax - 2} = \sqrt{x^2 - 2}$
 $x \in [2, \infty)$

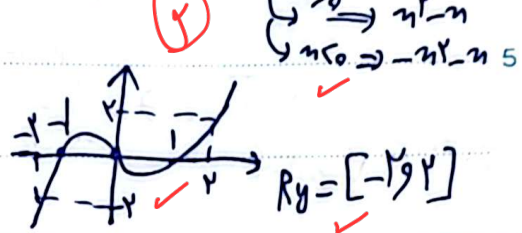
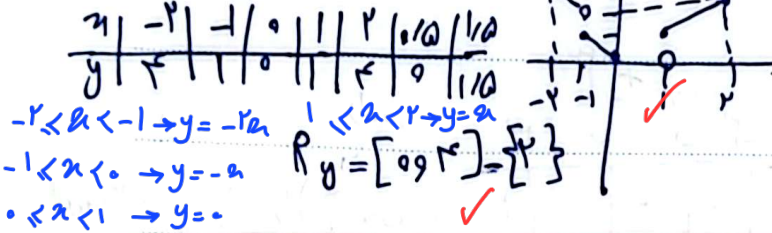
$$|x-2| - |x+1| = kx \quad \begin{cases} x > 2 \rightarrow 2x - x + 1 = 1 \\ 1 < x < 2 \rightarrow -x + 2 - x + 1 = -2x + 3 \\ x < 1 \rightarrow -x + 2 + x - 1 = 1 \end{cases}$$

$$f(x) = kx \quad f(2) = 1 \rightarrow 2k = 1 \rightarrow k = \frac{1}{2}$$

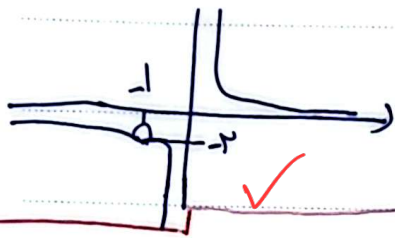


الف) $y = x[x]$ نمایش بازه های فراموش نشود

ب) $y = x/|x| - x$



$$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} = \frac{x^2+x}{x^2+x} = \frac{x}{x} \rightarrow x \neq -1$$



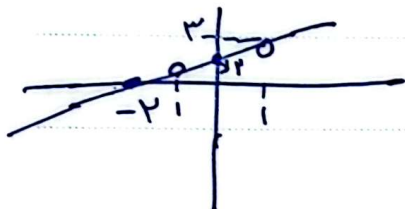
$$R_y = R - \{0, -1\} \rightarrow a+b = -1$$

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$$f(x) = \frac{x - \sqrt{x} + 3}{x - 2\sqrt{x} + 1} = \frac{(\sqrt{x}-1)(\sqrt{x}-3)}{(\sqrt{x}-1)^2} = \frac{\sqrt{x}-3}{\sqrt{x}-1}$$

مخرج $\sqrt{x}-1$ $\Rightarrow$ خارج $x \geq 1$ $\Rightarrow (-\infty, 1) \cup [3, +\infty) \cap R_f$

$$f(x) = \frac{x^3 + 2x^2 - x - 2}{x^2 - 1} = \frac{(x+1)(x+2)(x-1)}{(x-1)(x+1)} = x+2$$



$$R_f = R - \{-1, 2\} \rightarrow 1+2 = 3$$

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