۱

الف) $\left(x-2\left[\frac{x}{2}\right]\right)^2 \Rightarrow \left(2\left(\underbrace{\frac{x}{2}-\left[\frac{x}{2}\right]}_{[0,1)}\right)\right)^2 \Rightarrow 4\left(\frac{x}{2}-\left[\frac{x}{2}\right]\right)^2$

بین ۰ تا ۱ است ی دانیم برش

$\Rightarrow R_f=[0;4)$

ب) $\log y - \sin x = 4$

$\Rightarrow \log y = \sin x + 4 \Rightarrow y = 10^{4+\sin x}$, max: $10^{4+1}=y \Rightarrow y=10^5$, min: $10^{4-1}=y \Rightarrow y=10^3$, $R_f=[10^3,10^5]$

۲

$y=\sqrt{\frac{x^2-4}{x^2-9}} \xrightarrow[\text{عوض شود}]{y,x \text{ جای}} x=\sqrt{\frac{y^2-4}{y^2-9}} \Rightarrow x^2=\frac{y^2-4}{y^2-9} \Rightarrow x^2y^2-9x^2=y^2-4$

$\Rightarrow x^2y^2-y^2=9x^2-4 \Rightarrow (x^2-1)y^2=9x^2-4 \Rightarrow y^2=\frac{9x^2-4}{x^2-1} \Rightarrow y=\sqrt{\frac{9x^2-4}{x^2-1}}$

$\Rightarrow$ $-1 \quad -\frac{2}{3} \quad 0 \quad \frac{2}{3} \quad 1$ $\Rightarrow \left[0,\frac{2}{3}\right] \cup (1,+\infty) \Rightarrow \frac{5}{3}=a+b+c$

۳

الف) $f(x)=x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}}$ $\quad x+\frac{1}{x}\geq 2 \quad x>0$; $\quad x+\frac{1}{x}\leq -2 \quad x<0$

$\xrightarrow{x>0} x+\frac{1}{x}\geq 2 \Rightarrow 2\sqrt{x+\frac{1}{x}} \geq 2\sqrt{2} \Rightarrow$ عبارت $\geq 2+2\sqrt{2} \Rightarrow R_f=[2+2\sqrt{2},+\infty)$

$\xrightarrow{x<0} x+\frac{1}{x}\leq -2 \Rightarrow 2\sqrt{x+\frac{1}{x}} \Rightarrow$ حضی ندارد $\quad D_f \Rightarrow x>0$

ب) $(3-\sin x)(1+\sin x) \Rightarrow -\sin^2 x+2\sin x+3 \geq 0 \Rightarrow -(\sin x-1)^2+4=0$

$\sin x-1 \Rightarrow R=[-2,0] \Rightarrow -(\sin x-1)^2 = R_g=[-4,0] \Rightarrow \boxed{R_y=[0,4]}$

۴

$\left(-\frac{1}{3}+10\right)+\left(-\frac{2}{3}+10\right)+\dots+\left(-\frac{9}{3}+10\right)$ $\quad \boxed{y=-\frac{1}{3}x+10}$

$=(9\times 10)-(1+2+\dots+9)\left(\frac{1}{3}\right)=90-\frac{45}{3}=90-15=75$

۵

$f(x)=\sqrt{ax^2+bx+c} \Rightarrow$ چون در بازه $(2,+\infty)$ تعریف می شود ریشه آن ۲ است

$\Rightarrow ax^2+bx+c = k(x-2)^2 = k(x^2-4x+4)$

$\xrightarrow{x=3} k(\underbrace{9-4(3)+4}_{1})=1 \Rightarrow k=1 \Rightarrow a=1,\ b=-4,\ c=4$

$\Rightarrow g(x)=\sqrt{-4x^2+x-8} \Rightarrow \Delta=b^2-4ac=1-4(-8)(-4)<0$, $a<0$ $\Rightarrow$ برد ندارد $\varnothing$

$|x-2| - |x-1| = kx$ ۶

$kx \Rightarrow (1,1)\ (k=1)$

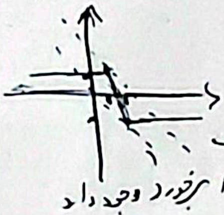
$\Rightarrow (2,-1) \Rightarrow \left(k=-\frac{1}{2}\right)$

$1 \quad 2$

$1 \quad | \quad -2x+3 \quad | \quad -1$

$-x+2+x-1=1 \qquad x-2-x+1=-1$

$-x+2-x+1=-2x+3$

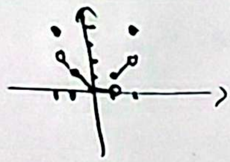


تنها حالتی که احتمال ۲ برخورد وجود دارد

الف) ۷

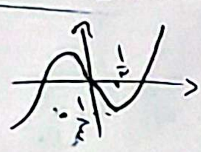
$y = x[x] \qquad R_f = [0,2)\cup\{4\}$

$[-2,-1]\ ,[-1,0]\ ,[0,1]\ ,[1,2]$



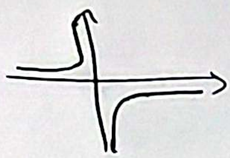
ب)

$y = x|x| - x \qquad \begin{cases} x^2-x & x\geq 0 \\ -x^2-x & x<0 \end{cases} \qquad R_f = R$



$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} \Rightarrow y = \frac{(x+1)+(x)+1}{x^2+x} = \frac{2\cancel{(x+1)}}{x\cancel{(x+1)}}$ ۸

$= \frac{2}{x}$



$R_f = R-\{0\}$

$\frac{x-4\sqrt{x}+3}{x-2\sqrt{x}+1} \Rightarrow \sqrt{x}=t \Rightarrow \frac{t^2-4t+3}{t^2-2t+1} \Rightarrow x\geq 0 \Rightarrow y=(-1,+\infty)$ ۹

$\sqrt{y} = \frac{(x-3)^2}{(x-1)^2}$

$\frac{\cancel{(t-1)}(t-3)}{\cancel{(t-1)}(t-1)} = \frac{\sqrt{x}-3}{\sqrt{x}-1} = y \Rightarrow \frac{\sqrt{y}-3}{\sqrt{y}-1} = x \Rightarrow \sqrt{y}\,x - x = \sqrt{y}-3$

$\Rightarrow \sqrt{y}(x-1) = x-3$

$R_f = [-1,+\infty)$

$f(x) = \frac{x^3+2x^2-x-2}{x^2-1} \Rightarrow \frac{x^2(x+2)-1(x+2)}{x^2-1} = \frac{\cancel{(x^2-1)}(x+2)}{\cancel{(x^2-1)}}$ ۱۰

$= x+2 \Rightarrow \begin{cases} x=1 \Rightarrow x+2=3 \\ x=-1 \Rightarrow x+2=+1 \end{cases} \Bigg\} = 4$