

الف) $y = (\sqrt{x - [x]})^2 \xrightarrow{\frac{x}{y} = t} y = x(t - [t])^2$

$0 \leq t - [t] < 1 \rightarrow 0 \leq (t - [t])^2 < 1 \rightarrow 0 \leq x(t - [t])^2 < x$

$R_f = [0, x)$

ب) $\log y = \sin x + x \rightarrow y = 10^{\sin x + x}$

$-1 \leq \sin x \leq 1 \rightarrow x \leq \sin x + x \leq x + 1$

$10^{-1} \leq \sin x + x \leq 10^1 \rightarrow 10^{-1} \leq \sin x + x \leq 10$

$R_f = [10^{-1}, 10]$

$y = \frac{\sqrt{x^2 - 4}}{x^2 - 4} \rightarrow \frac{x^2 - 4}{x^2 - 4} = y \rightarrow x^2 = \frac{-4y^2 + 4}{y^2 - 1} = \frac{4y^2 - 4}{y^2 - 1}$

$x = \pm \sqrt{\frac{4y^2 - 4}{y^2 - 1}} \quad \frac{4y^2 - 4}{y^2 - 1} \geq 0 \rightarrow \frac{(y-1)(y+1)}{(y-1)(y+1)} \geq 0 \Rightarrow \frac{-1}{+} \frac{-\frac{1}{y}}{-} \frac{\frac{1}{y}}{+} \frac{1}{+}$

$R_f = (-\infty, -1) \cup [-\frac{1}{y}, \frac{1}{y}] \cup (1, +\infty)$

$a + b + c = -\frac{1}{y} + \frac{1}{y} + 1 = 1$

الف) $f(x) = x + \frac{1}{x} + \sqrt{x + \frac{1}{x}} = t + \sqrt{t}$

$t \rightarrow (-\infty, -2) \cup (2, +\infty)$

$t \geq 0 \rightarrow [2, +\infty)$

$\max t = +\infty \rightarrow f(x) = +\infty$

$\min t = 2 \rightarrow f(x) = 2 + \sqrt{2}$

$-\frac{1}{x} = -1 \times$

$R_f = [2 + \sqrt{2}, +\infty)$

ب) $f(x) = 3 + 3\sin x - \sin x - \sin^2 x = -\sin^2 x + 2\sin x + 3 = -z^2 + 2z + 3$

$-1 \leq z \leq 1$

$\max z = 1 \rightarrow f(x) = 4$

$\min z = -1 \rightarrow f(x) = 0$

$-\frac{1}{x} = -\frac{1}{-1} = 1 \rightarrow f(x) = 4$

$R_f = [0, 4]$

$D_f \rightarrow R_f$

$1 \rightarrow \frac{1}{x}$

$2 \rightarrow \frac{1}{x}$

$3 \rightarrow \frac{1}{x}$

$4 \rightarrow \frac{1}{x}$

$5 \rightarrow \frac{1}{x}$

$6 \rightarrow \frac{1}{x}$

$7 \rightarrow \frac{1}{x}$

$8 \rightarrow \frac{1}{x}$

$9 \rightarrow \frac{1}{x}$

$\sum_{i=1}^9 \frac{(x_i + 1) + \dots + (x_i + 9)}{9} = \frac{9 \times 1 + (1 + \dots + 9)}{9} = 3 \times 1 + 1 \times 9 = 10$

$f(x) = \frac{y}{-x + 1} \rightarrow a = 0 \rightarrow f(x) = \sqrt{bx + c}$

$f(2) = \sqrt{2b + c} = 1 \rightarrow 2b + c = 1$

$f(4) = \sqrt{4b + c} = 0 \rightarrow 4b + c = 0$

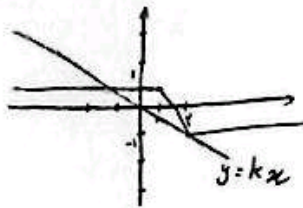
$b = 1 \quad c = -2$

$g(x) = \sqrt{bx^2 - 2c} = \sqrt{x^2 + 4}$

$y = bx^2 - 2c \rightarrow \text{ext } \sqrt{bx^2 - 2c} = \sqrt{4} = 2$

$R_g = \sqrt{[2, +\infty)} = [\sqrt{2}, +\infty)$

$$y = |x-2| - |x-1|$$

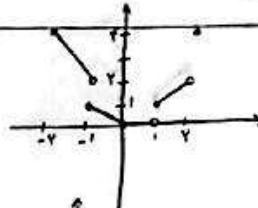


$$y = kx \rightarrow (0,0) \quad (2,-1) \rightarrow k = \frac{-1-0}{2-0} = -\frac{1}{2}$$

$$y = -\frac{1}{2}x$$

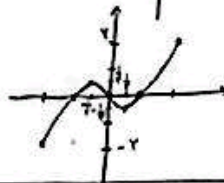
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الف) $f(x) = \begin{cases} -2x & -2 \leq x < -1 \\ -x & -1 \leq x < 0 \\ 0 & 0 \leq x < 1 \\ x & 1 \leq x < 2 \\ 2x & x = 2 \end{cases}$



$$R_f = \mathbb{R} - \{2\}$$

ب) $f(x) = \begin{cases} x^2 - x & x \geq 0 \\ -x^2 - x & x \leq 0 \end{cases}$

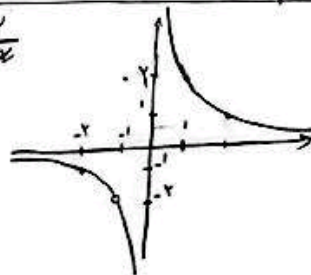


$$R_f = [-2, 2]$$

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$$y = \frac{2x+2}{x^2+x} = \frac{2}{x}$$

$$D_f = \mathbb{R} - \{-1, 0\}$$



$$R_f = \mathbb{R} - \{-2, 0\}$$

$$a+b = -2+0 = -2$$

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$$f(x) = \frac{(\sqrt{x}-1)(\sqrt{x}-2)}{(\sqrt{x}-1)(\sqrt{x}-1)} = \frac{\sqrt{x}-2}{\sqrt{x}-1} = y \rightarrow \sqrt{x} = -\frac{-y+2}{y-1} = \frac{y-2}{y-1}$$

$$D_f = \mathbb{R} - \{1\}$$

$$\sqrt{x} = \frac{y-2}{y-1}$$

$$x = \left(\frac{y-2}{y-1}\right)^2 \rightarrow \mathbb{R} - \{1\}$$

$$R_f' = [0, +\infty) - \{1\}$$

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$$D_f = \mathbb{R} - \{-1, 1\}$$

$$f(x) = \frac{x^2+2x^2-x-2}{x^2-1} = \frac{(x+2)(x^2-1)}{x^2-1} = x+2$$

خزات $\rightarrow x=1 \rightarrow y=3$

$$x+1=3$$

$\rightarrow x=-1 \rightarrow y=1$

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