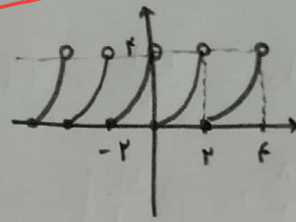


14, 5

الف) $y = \left(x - 2 \left\lfloor \frac{x}{2} \right\rfloor \right)^2 = \begin{cases} (x+2)^2 & -2 \leq x < 0 \\ x^2 & 0 \leq x < 2 \\ (x-2)^2 & 2 \leq x < 4 \end{cases}$



$R_f = [0, 4]$

ب) $-\sin x + \log y - t = 0 \quad \log y = \sin x + t \rightarrow y = 10^{\sin x + t}$

$-1 \leq \sin x \leq 1 \xrightarrow{+t} -1+t \leq \sin x + t \leq 1+t \xrightarrow{\text{الگوی صعودی}} 10^{-1+t} \leq 10^{\sin x + t} \leq 10^{1+t} \rightarrow R_f = [10^{-1+t}, 10^{1+t}]$

$f(x) = \sqrt{\frac{x^2-4}{x^2-9}} \rightarrow y = \sqrt{\frac{x^2-4}{x^2-9}} \xrightarrow{y \geq 0} y^2 = \frac{x^2-4}{x^2-9} \rightarrow y^2 x^2 - 9y^2 = x^2 - 4$

$\rightarrow y^2 x^2 - x^2 = 9y^2 - 4 \rightarrow x^2 = \frac{9y^2 - 4}{y^2 - 1} \xrightarrow{\sqrt{\quad}} x = \pm \sqrt{\frac{9y^2 - 4}{y^2 - 1}}$

$D_{f^{-1}} = R_f \rightarrow \frac{9y^2 - 4}{y^2 - 1} \geq 0 \rightarrow \frac{-1 - \frac{2}{3} \quad \frac{2}{3} \quad 1}{+ \quad - \quad + \quad - \quad +} \cap y \geq 0 \rightarrow [0, \frac{2}{3}] \cup (1, +\infty)$

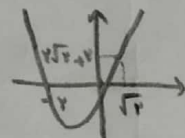
$\xrightarrow{+} 0 + \frac{2}{3} + 1 = \frac{5}{3}$

این نمودار $y = t^2 + 2t$ است $f(x)$

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}}$

$\sqrt{x + \frac{1}{x}} = t$

$y = t^2 + 2t \quad t \geq \sqrt{2}$



$R_f = [2\sqrt{2} + 2, +\infty)$

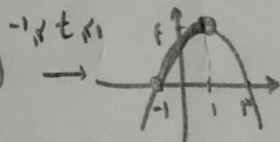
$x + \frac{1}{x} \geq 0 \rightarrow x > 0 \rightarrow x + \frac{1}{x} \geq 2 \rightarrow \sqrt{x + \frac{1}{x}} \geq \sqrt{2} \quad ①$

راه دوم: $x^2 + 2 \geq 0$ بسیار آسان، بیشترین

ب) $y = (3 - \sin x)(1 + \sin x)$

$\sin x = t$

$y = (3 - t)(1 + t)$



$R_f = [0, 4]$

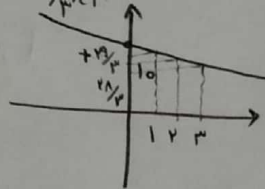
راه دوم: $-\sin^2 x + 2\sin x + 3 \rightarrow \sin x = t$

$t = -1, y = 0$

$t = 1, y = 4 \quad R_f = [0, 4]$

$\frac{-0}{2 \times 1} = 1, y = 4$

$$f(x) = -\frac{1}{3}x + 10, \quad D_f = \{1, 2, \dots, 9\}$$



1, 10

$$f(1) = -\frac{1}{3} + 10 = \frac{29}{3}, \quad f(2) = -\frac{2}{3} + 10 = \frac{28}{3}, \quad f(9) = -\frac{9}{3} + 10 = \frac{21}{3}$$

$$R_f = \left\{ \frac{29}{3}, \frac{28}{3}, \dots, \frac{21}{3} \right\}$$

مجموع اعضای برد = $\sqrt{a}$

سپس ما همواره برای دامنه‌ی آشنا زیرادیکال دارای دو بازه یا $\mathbb{R}$ یا $\{0\}$ یا $\emptyset$ می باشند و دارای یک بازه نیستند.

$$\textcircled{1} \rightarrow a=0 \rightarrow y = \sqrt{bx+c}$$

$$D_f = [2, +\infty) \xrightarrow{f(2)=0} 2b+c=0 \quad \textcircled{2}$$

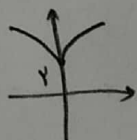
$$\xrightarrow{\textcircled{2}, \textcircled{3}} b=1, c=-2$$

$$f(3)=1 \rightarrow 3b+c=1 \quad \textcircled{3}$$

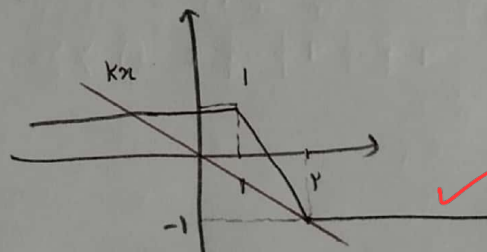
$$\rightarrow f(x) = \sqrt{x-2}$$

$$g(x) = \sqrt{x^2 + f}$$

$$\frac{-b}{2a} = 0 \rightarrow f(0) = f \rightarrow \sqrt{f(0)} = 2, \quad R_g = [2, +\infty)$$



$$|x-2| - |x-1| = kx$$



$$f(2) = -1 \rightarrow 2k = -1 \rightarrow k = -\frac{1}{2}$$

$$\textcircled{a) } y = x[x] = \begin{cases} -2x \\ -x \\ 0 \\ x \\ 2 \end{cases}$$

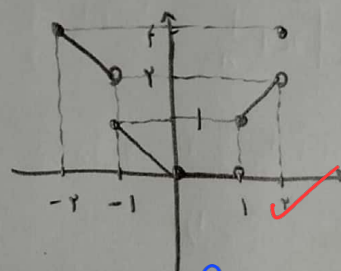
$$-2, 2x < -1$$

$$-1, 2x < 0$$

$$0 \leq x < 1$$

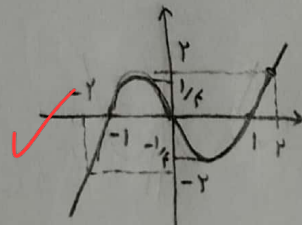
$$1 \leq x < 2$$

$$x=2$$



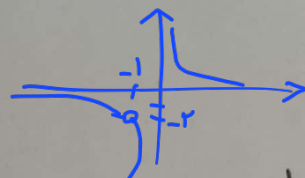
$$R_f = [0, 2] - \{2\}$$

$$\textcircled{b) } y = x|x| - x = \begin{cases} x^2 - x & x > 0 \\ -x^2 - x & x < 0 \end{cases}$$



$$R_f = [-2, 2]$$

$$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^r + x} = \frac{y(x+1)}{x^r + x} = \frac{y(x+1)}{x(x+1)} = \frac{y}{x}$$



-1

(1, 2)

$$y = \frac{y(x+1)}{x^r + x} \rightarrow yx^r + yx = y(x+1) \rightarrow yx^r + (y-r)x - r = 0, \quad x = \frac{-b \pm \sqrt{(y-r)^2 + 4y}}{2a}$$

$$D_{f^{-1}} = R_f \rightarrow (y-r)^2 + 4y \geq 0 \rightarrow y^2 + r + 4y \geq 0 \rightarrow (y+r)^2 \geq 0 \rightarrow R$$

$$R_f = R - \{-r, 0\}$$

$$R_f = R - \{-r, r\} \rightarrow a+b=0$$

$$a+b = -r$$

$$\lim_{x \rightarrow 0} \frac{y(x+1)}{x^r + x} \stackrel{\text{hop}}{=} \frac{y}{y(x+1)} \stackrel{\text{hop}}{=} \frac{1}{x+1} = 1$$

$$\lim_{x \rightarrow -1} \frac{y(x+1)}{x^r + x} \stackrel{\text{hop}}{=} \frac{y}{y(x+1)} = -r$$

$$f(x) = \frac{x - r\sqrt{x} + r}{x - r\sqrt{x} + 1} = \frac{(\sqrt{x}-r)(\sqrt{x}-1)}{(\sqrt{x}-1)^2} \stackrel{x \neq 1}{=} \frac{\sqrt{x}-r}{\sqrt{x}-1} = R_f = (-\infty, 1) \cup [r, +\infty)$$

-9

(r)

$$f(x) = \frac{(x-1)(x^r + r(x+1))}{(x-1)(x+1)} \stackrel{x \neq 1}{=} \frac{x^r + r(x+1)}{x+1} = \frac{(x+1)(x-1)(x+r)}{(x+1)(x-1)} = x+r$$

$$f(1) = \frac{r}{1} \text{ ①}$$

$$x \neq \pm 1$$

$$R - \left\{ \frac{r}{1}, 1 \right\} \xrightarrow{\text{①} + \text{②}} \frac{r}{1} + \frac{r}{1} = \frac{2r}{1}$$

$$\lim_{x \rightarrow -1} \frac{x^r + r(x+1)}{x+1} \stackrel{\text{hop}}{=} \frac{r(x+1)}{1} = r \text{ ②}$$

$$R_f = R - \{1, r\}$$

$$a+b = r$$