① الف)
$y=(x-2[\frac{x}{2}])^2=4(\frac{x}{2}-[\frac{x}{2}])^2$
$\Rightarrow 0\le\frac{x}{2}-[\frac{x}{2}]<1 \Rightarrow 0\le(\frac{x}{2}-[\frac{x}{2}])^2<1 \Rightarrow$
$0\le 4(\frac{x}{2}-[\frac{x}{2}])^2<4 \quad R=[0,4)$

ب)
$-\sin y+\log y-4=0 \Rightarrow \log y=\sin y+4 \Rightarrow$
$-1\le\sin y\le 1 \Rightarrow 3\le\sin y+4\le 5 \Rightarrow 2\le\log y\le 5 \Rightarrow$
$\log_{10}10^2\le\log y\le\log_{10}10^5 \Rightarrow y=[10^2,10^5] \Rightarrow R=[10^2,10^5]$

② $f(x)=\sqrt{\frac{x^2-4}{x^2-9}}$
$x^2=0 \Rightarrow y=f(x)=\sqrt{\frac{4}{9}}=\frac{2}{3}$ $\quad x^2=+\infty \Rightarrow y=f(x)=\sqrt{1}=1$
چون خروج صفر می‌شود $\Rightarrow (0,\frac{2}{3}]\cup(1,+\infty) \Rightarrow \begin{cases}a=0\\b=\frac{1}{3}\\c=1\end{cases}$
$a+b+c=0+\frac{1}{3}+1=\frac{4}{3}$

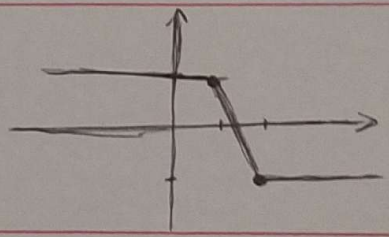
③ الف)
$f(x)=x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}}$ $\quad x+\frac{1}{x}\Rightarrow R=[2,+\infty)$
$R=[2+2\sqrt{2},+\infty)$

ب)
$y=(2-\sin x)(1+\sin x)=-\sin^2x+\sin x+2$
$\sin x=1\Rightarrow y=2$ $\quad \sin x=-1\Rightarrow y=0$ $\quad -\frac{b}{2a}=\frac{1}{2}\Rightarrow y=\frac{9}{4}$
$R=[0,\frac{9}{4}]$

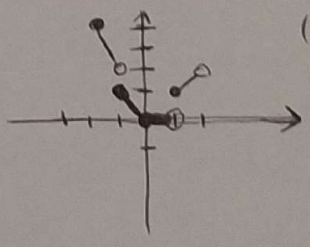
④ $f(x)=-\frac{1}{3}x+10$ $\quad D_f=\{1,2,3,4,5,6,7,8,9\}$
$f(1)+f(2)+f(3)+f(4)+f(5)+f(6)+f(7)+f(8)+f(9)=$
$=\frac{29}{3}+\frac{28}{3}+\frac{27}{3}+\frac{26}{3}+\frac{25}{3}+\frac{24}{3}+\frac{23}{3}+\frac{22}{3}+\frac{21}{3}=\frac{225}{3}=75$

⑤ $f(x)=\sqrt{ax^2+bx+c}$ $\quad D_f=[2,+\infty)$
$ax^2+bx+c\Rightarrow$ یک معادله خط است $\Rightarrow a=0, b>0$
$f(2)=0=\sqrt{2b+c}=0\Rightarrow 2b+c=0$ $\quad f(3)=1=\sqrt{3b+c}=1\Rightarrow 3b+c=1$
$\begin{cases}2b+c=0\\3b+c=1\end{cases}\Rightarrow b=1, c=-2$
$g(x)=\sqrt{bx^2+ax-2c}=\sqrt{x^2+4} \Rightarrow R_g=[2,+\infty)$

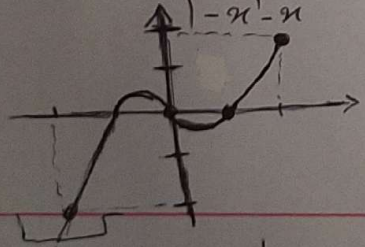
⑥ $|x-2|-|x-1|=kx$ $\quad |x-2|-|x-1|$ تابع سرراهی است
$kx\Rightarrow$ با وجود این که مختصات عبور کند
$x=2\Rightarrow y=-1=k(2)\Rightarrow 2k=-1\Rightarrow$
$\Rightarrow k=-\frac{1}{2}$



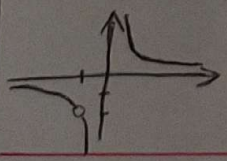
⑦ الف)
$y=x[x]$
$R=[0,4]-\{2\}$



ب)
$y=x|x|-x \Rightarrow \begin{cases}x^2-x & x\ge 0\\ -x^2-x & x<0\end{cases}$
$R=[-2,2]$



⑧ $y=\frac{1}{x}+\frac{1}{x+1}+\frac{1}{x^2+x}$ $\quad R=\mathbb{R}-\{a,b\}$
$\frac{1}{x}+\frac{1}{x+1}+\frac{1}{x^2+x}=\frac{2x+2}{x^2+x}=\frac{2(x+1)}{x(x+1)}=\frac{2}{x}$
$R=\mathbb{R}-\{a,b\}=\mathbb{R}-\{0,-2\}$ $\quad a+b=-2+0=-2$



⑨ $f(x)=\frac{x-4\sqrt{x}+3}{x-2\sqrt{x}+1}=\frac{(\sqrt{x}-1)(\sqrt{x}-3)}{(\sqrt{x}-1)(\sqrt{x}-1)}=\frac{\sqrt{x}-3}{\sqrt{x}-1}$
$x=0\to y=\frac{-3}{-1}=3$ $\quad x=+\infty\to y=1$
چون خروج صفر می‌شود $\Rightarrow R_f=(-\infty,1)\cup[3,+\infty)$

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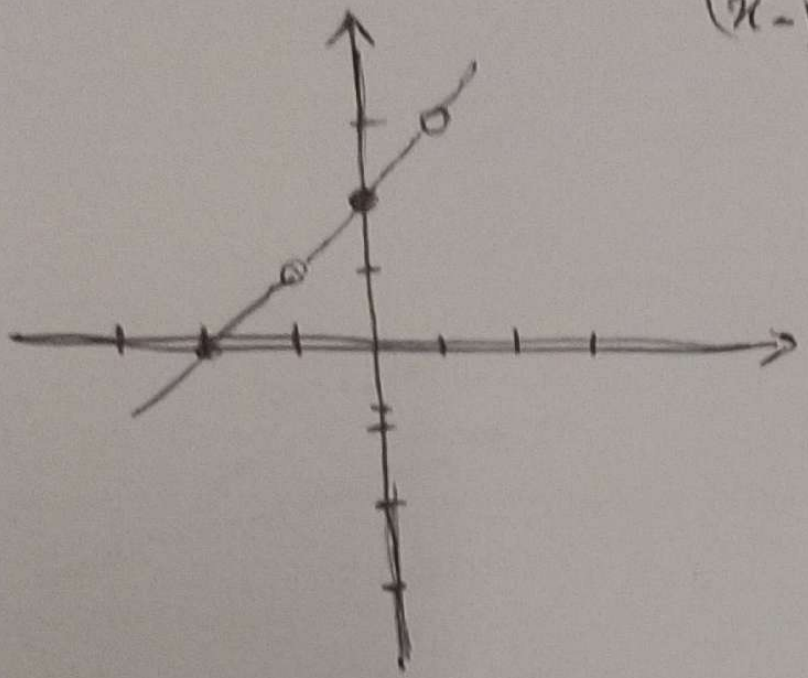
(10)

$$f(x) = \frac{x^3 + 2x^2 - x - 2}{x^2 - 1} = \frac{(x-1)(x^2 + 3x + 2)}{(x-1)(x+1)} = \frac{(x+1)(x+2)}{(x+1)} = x + 2$$



$R_f = \mathbb{R} - \{1, 2\}$

$1 + 2 = 3$

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